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AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

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Resolving Time of a Binary Counter: A Laboratory Exercise

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(Received January 28, 1953)

This nonquantitative electronic circuit exercise is suitable for beginning or advanced students. The minimum operating time of a binary counter is measured in microseconds by means of a twin pulse generator. The operating cycle of the latter is studied in terms of its oscilloscope patterns.

IN this exercise the senior or first-year graduate student is asked to interpret a series of oscilloscope patterns in terms of the function and time sequence of voltage wave forms occurring in a complicated electronic device. The consideration of functioning in terms of circuit components, i.e., the technology of the circuit, is kept to a minimum that is within the grasp of a student with only elementary training in electronics. The problem set for the student is to understand at the "block diagram level" how an integrated system of vacuum tubes operates to produce a desired end result. The circuit behavior is sufficiently unusual and complex that even the more advanced student will find it worthwhile.

From two years of use it can be said that the intellectual factors which are measured, and presumably developed, by this experiment are distinguishable from those contributing generally to scholastic excellence. After allowance for background it is still strikingly apparent that some "A" students do poorly on this equipment compared to others who are reputedly mediocre in their physics courses.

The present exercise is unorthodox in that there are no conventional dial readings to take. Most teaching laboratory experiments require the recording of quantitative data and their subsequent exhibition or manipulation. With the assistance of a well-written manual this procedure may degenerate into the cookbook gathering of figures. The exasperated laboratory instructor is then tempted to distrust Lord Kelvin's dictum: "When you can measure what you are speaking about and express it in numbers you know something about it." On the other hand, if no numbers are provided behind which the student can take refuge, perhaps his obligation to understand will appear inescapable. Such, at least, is the plausible hypothesis around which this experiment has been built.

Before describing the equipment and the manner of its presentation to the student, its main features will be mentioned. The experiment involves the measurement in microseconds of the resolving time of a scaling circuit. A twin pulse generator must be built, which becomes a useful research tool in its own right. The parts cost is under 50 dollars. The only other items required are a triggered-sweep video oscilloscope and a regulated power supply.

In the generation of periodic twin pulses with variable spacing, all the common pulse-handling circuits are used: astable and bistable multivi-

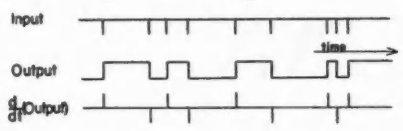

FIG. 1. Division of input impulses by a factor of two by means of a bistable multivibrator.

brators, saw-tooth generator, blocking oscillator, coupling diodes, and cathode follower. Primarily, the student is training his mind to see logical relationships among unfamiliar space-time concepts, while secondarily, he is acquiring a functional familiarity with circuits of great technical importance.

PRESENTATION TO THE STUDENT

The response to the experiment will depend upon the kind and amount of information given to the student. Before the laboratory period the power supply and oscilloscope are interconnected with the twin-pulse generator so that nothing remains for the student but to adjust the controls and watch the patterns on the face of the oscilloscope. The student is given the oscilloscope manufacturer's instruction book together with a specially prepared experiment instruction manual, and is thereafter left alone as much as possible.

The manual written for this experiment begins with a narrative discussion of the uses of scaling circuits for counting purposes in industry and research. Counter resolving time is defined as the minimum spacing between successive impulses that will allow them to be counted separately.

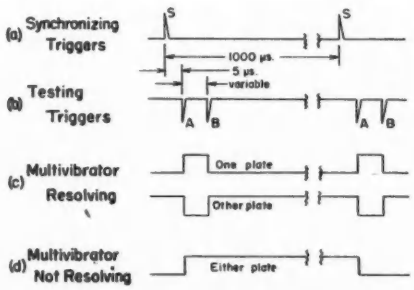

FIG. 2. Use of paired test triggers to determine the minimum operating time of a multivibrator in microseconds.

Next, the bistable multivibrator is defined in functional terms and introduced as the basic element of most scaling circuits. The circuit of the plate-coupled bistable multivibrator is presented in simplified form and analyzed qualitatively. Its scale-of-two action is described by a time-amplitude diagram (Fig. 1) showing randomly spaced input triggers, or impulses, which generate square waves, the latter being differ-

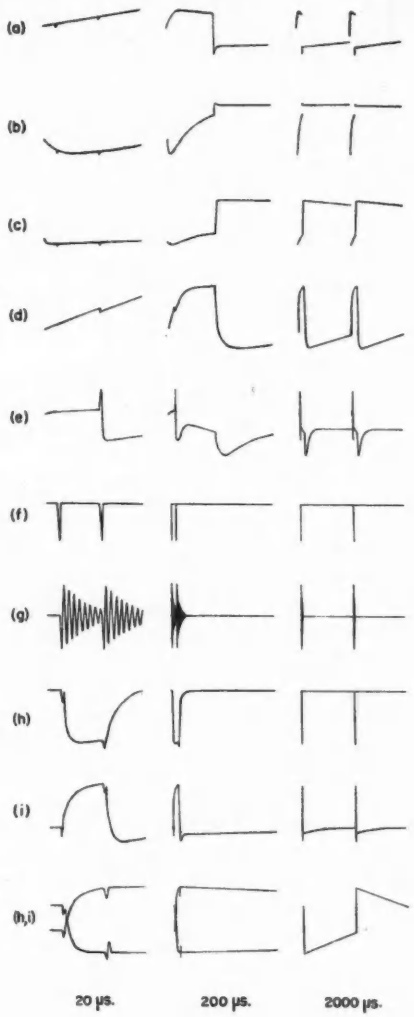

FIG. 3. Actual wave forms of twin-pulse generator as seen at the correspondingly lettered terminals of Figs. 4 and 5 for three oscilloscope sweep lengths.

entiated to yield half as many output as input triggers. The factors determining speed of multivibrator operation are discussed briefly.

The twin-pulse generator is described at three levels of complexity. First, in terms of outputs and controls as follows: There are two main output connections delivering triggers as shown in Fig. 2a, b. The entire pattern repeats every 1000 microseconds. The synchronizing trigger *S*, used to start the viewing oscilloscope sweep, occurs about five microseconds before trigger *A*, with which the actual testing operation begins. Twin triggers *A* and *B* are fed to the scaling circuit under test. If the scaling circuit operates (counts) once for trigger *A* and again for trigger *B*, the triggers are said to be "resolved." The spacing between *A* and *B* can be varied from zero to 15 microseconds, and in the course of the test is adjusted to the minimum value that will allow the triggers to be separately counted.

Although the test triggers are available for external use, they are also coupled to a bistable plate-coupled multivibrator which is a permanent part of the twin-pulse generator. It is the resolution time of this multivibrator that is to be measured by the student.

The idealized voltage wave forms of this kind of multivibrator have been explained by Fig. 1 in terms of well-spaced random triggers. The corresponding idealized wave forms (Fig. 2c, d) to be expected with the given pattern of test triggers are not shown in the instruction manual. The student must visualize these for himself before he can hope to recognize the actual wave forms on the oscilloscope. The actual wave forms (Fig. 3f, h, i) appear before the student as he turns the view-selector switch and adjusts the trigger spacing knob and the oscilloscope controls.

When the spacing of the twin triggers has been adjusted so that they can just be resolved, that spacing is measured in the following way. The wave-form view-selector switch is thrown to position *g*, which connects a capacitor-inductor combination that has been previously tuned to one megacycle. The tuned circuit is shock-excited by the twin triggers and the resolving time of the multivibrator determined by counting the number of cycles between triggers (Fig. 3g). The resolving time for the multivibrator design of Fig. 5 is about eight microseconds.

An understanding of the manual material discussed up to this point will prepare even the more inexperienced student to reach the major objectives of the experiment. At the second level of complexity the manual shows the block diagram of Fig. 4, and the internal functioning of the twin-pulse generator is described in terms of of the idealized wave forms appearing on that block diagram.

The astable multivibrator runs freely, controlling the entire testing operation. Once every millisecond it generates a square wave 90 microseconds long which includes the period of actual testing. The leading edge of this square wave is differentiated to provide synchronizing triggers of either polarity for initiating the sweep in the external oscilloscope. This same square wave is fed to two saw-tooth generators whose outputs

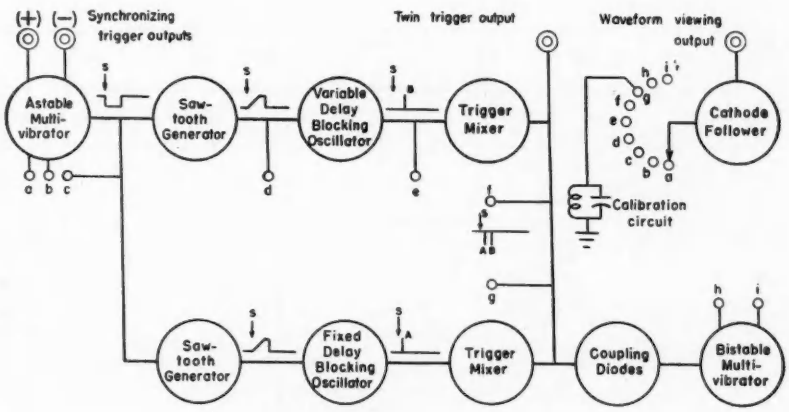

FIG. 4. Functional diagram of twin-pulse generator.

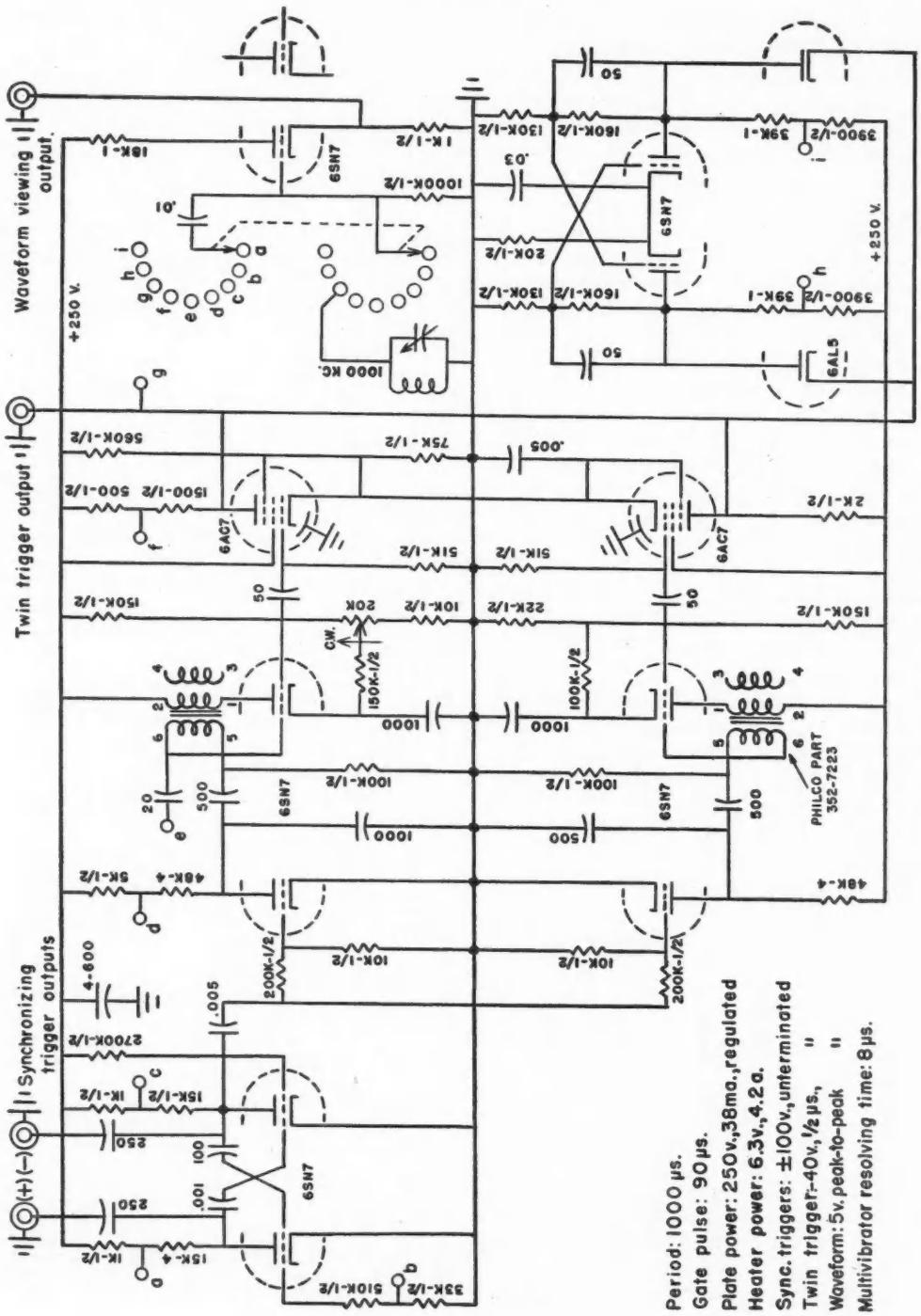

FIG. 5. Circuit diagram of twin-pulse generator.

consist of a voltage which rises more or less linearly with time for 30 microseconds. A blocking oscillator is triggered by each saw-tooth generator at a point where the rising saw-tooth voltage matches a pre-set dc bias on the oscillator. This firing bias, and therefore the delay time from the beginning of the square wave, is made fixed for the case of one blocking oscillator and variable for the other. The firing of the blocking oscillators generates the testing triggers, one fixed in time and one variable. These are mixed and used externally or fed internally to the bistable multivibrator for testing it.

Up to this point there is still no electronic circuitry for the student to understand except for the basic scale-of-two multivibrator. For the more able or experienced student the complete circuit diagram, Fig. 5, is shown in an appendix.

The mechanical layout of the twin-pulse generator follows approximately the layout of Fig. 5. The 13×17-inch chassis skeleton is rack-mounted in a vertical plane with all components exposed on a reversible plate.

The instructions specifically given to the student for the performance of this experiment are these:

1. Familiarize yourself with all scope and twin-pulse generator controls.
2. Observe, sketch, and understand in terms of idealized wave forms, each actual wave form visible on the 20 and 200 microsecond trigger-released sweeps and on a synchronized, free running sweep 2000 microseconds long.
3. Measure the resolving time of the built-in bistable multivibrator.
4. Observe the increase of resolving time due to increased stray capacitance when a three-foot laboratory test lead is clipped on to a plate of the bistable multivibrator.

A separate appendix of the manual gives a list of wave-form view-selector switch positions, describing in idealized form the patterns to be seen in each position and their functional meaning. Although this list repeats information given in the body of the manual, it is found to be a worthwhile convenience by the student when trying to understand the meaning of strange patterns.

The actual wave forms as sketched from the oscilloscope in Fig. 3 are not shown in the instruction manual. The student is warned that the actual wave forms will differ markedly from the idealized forms. The student's task, given the randomly triggered idealized wave forms of a bistable multivibrator (Fig. 1), is to visualize the idealized wave forms for the test operation as an endless function of time, to transform mentally these idealized wave forms to the form they would take on a synchronized oscilloscope with several differing sweep lengths, and to recognize the actual wave forms in relation to the idealized forms while simultaneously adjusting the oscilloscope controls.

One lesson learned indirectly by the student in this experiment is the practical significance of order-of-magnitude. As shown in Fig. 3, the correlation of patterns with a time-scale difference of one order of magnitude requires careful attention, while the direct correlation of patterns differing by two orders of magnitude is almost impossible unless one already understands the full significance of the patterns. The student does not expect this effect and is in most cases confused by it. The psychological and practical emergence of new features with a change of order of magnitude is one of the important aspects of nature. An adequate appreciation of this fact is too often gained by the student of physics only after he has completed his academic studies.

Some Generalizations in Geometrical Optics Derived by a Convergence Method

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(Received December 2, 1952)

A simple but general treatment of reflection and refraction at spherical surfaces is based on properties of nodal points and the specification of wave fronts by their convergences. Rules for the location and description of images are justified and extended to the images formed by thin lenses, the positions of whose nodal points make apparent an analogy with single refracting surfaces. The formula for the change in convergence of a wave front proceeding in a homogeneous and isotropic medium is applied to describe the images of axial objects, to derive Newton's relation in generalized form, and to consider refraction by separated refracting surfaces of unspecified powers. Solutions to five numerical problems, obtained by methods advocated in the paper, are appended.

I. INTRODUCTION

FIGURE 1 is a two-dimensional representation of light which is associated with a point focus and traveling in a homogeneous and isotropic medium. The concentric circles represent wave fronts and the radii (referred to as "rays") show the directions in which portions of the wave fronts progress. It is convenient to distinguish between wave fronts converging towards and diverging from a focus by associating their curvatures with positive and negative signs, respectively. Figure 2, in which is represented a wave front, some associated rays and an appropriately drawn cross, serves as a mnemonic for this; and Fig. 3 shows that different wave-curvatures may be represented by sagittae corresponding to a fixed length $2y$ of chord.

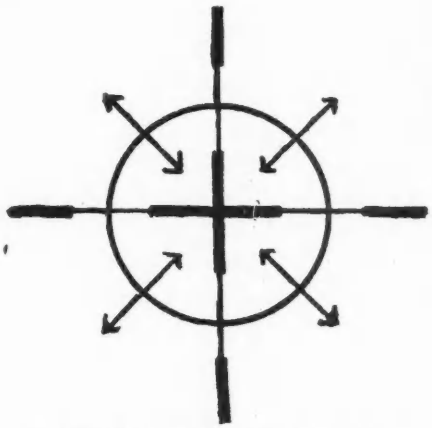


FIG. 2. Mnemonic for signs of curvatures (or convergences) and powers.

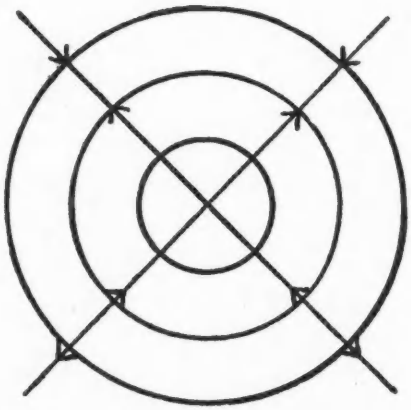


FIG. 1. Converging and diverging wave fronts and associated rays.

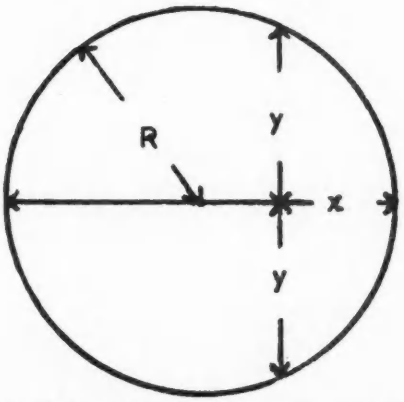


FIG. 3. The relation between curvature and sagitta. Since $y^2=(2R-x)x\doteq 2Rx$; curvature $=1/R=(2/y^2)$(Sag). In the diagrams which follow, y is kept constant and curvatures are represented by sags.

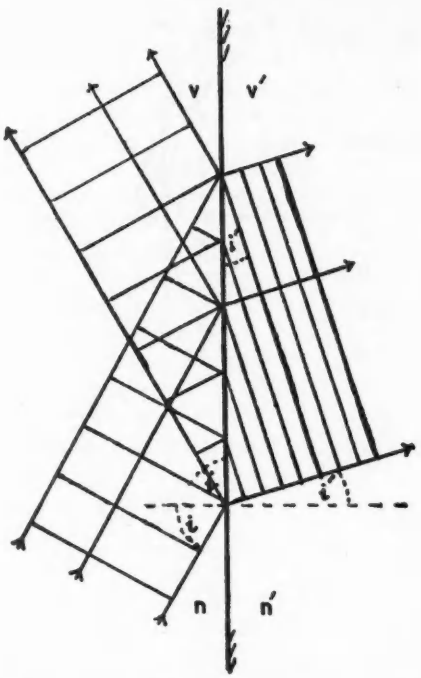


FIG. 4. Reflection and refraction of a plane wave at a plane surface. Note change in wavelength resulting from refraction.

In a homogeneous medium the curvature of a wave front varies progressively as the front advances, whereas at a reflecting or refracting surface there is, in general, a discontinuous change in curvature: these changes are considered in Secs. III and II, respectively.

II. DISCONTINUOUS CHANGES IN CURVATURE (OR CONVERGENCE) OF WAVE FRONTS BY REFLECTION AND REFRACTION

When a plane wave meets a plane surface, an application of Huygens' principle shows that the refracted and reflected wave fronts are also plane, and, if the wave front is inclined to the surface (Fig. 4), $\sin i/\sin i'$, defined as the relative refractive index, may be interpreted as $v/v' = (v_0/v')(v/v_0) = n'/n$, where v represents velocity, the subscript $_0$ refers to a vacuum, and n and n' are (absolute) refractive indices. In the more general case when the wave front diverges from O (Fig. 5), the portion incident at A traverses q with velocity v' in the same time that the portion incident at B traverses p with velocity v. Subject to the usual assumptions it is readily deduced that $-n/OA = -n'/IA$, the negative signs implying diverging wave fronts. Figure 6(a) shows, with a similar notation, the changes in curvature which occur when a plane wave is incident on a converging surface of small aperture and radius of curvature r, and Fig. 6(b) is the corresponding diagram for a diverging surface. By contrast Fig. 7 shows that when a wave has as focus N, the center of curvature of the surface upon which it is incident, no change occurs on reflection and refraction in the magnitude of the wave curvature at the surface, although, on reflection only, there is a change in sign.

A significant generalization results if, instead of specifying a wave front by its curvature, it is specified by its *convergence*, defined as the product

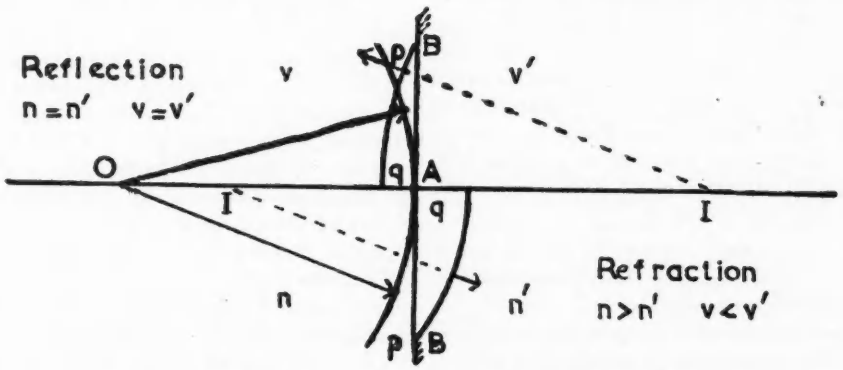


FIG. 5. Reflection and refraction of a diverging wave at a plane surface.
$q/v' = p/v$, therefore $-n'/IA = -n/OA$.

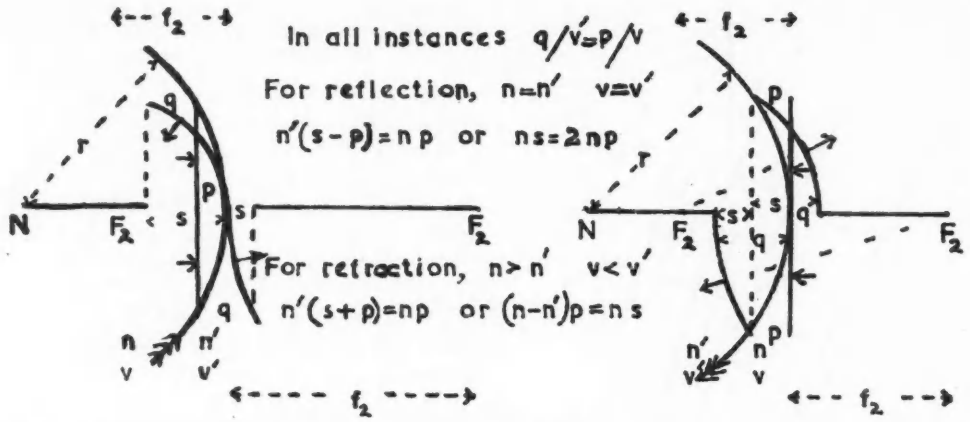


FIG. 6. Reflection and refraction of a plane wave by a spherical surface. Since s is the sag of the wave front associated with the principal focus F_2,

$$n/f_2 = 2n/r \qquad -n/f_2 = -2n/r$$

for reflection, and

$$n'/f_2 = (n-n')/r \qquad -n'/f_2 = -(n-n')/r$$

for refraction.

(a) Converging surface ($+ve$ power). (b) Diverging surface ($-ve$ power).

Point N represents the center of curvature of the surface and stands for nodal point. If light travels in a direction opposite to that indicated by the arrows, F_2 and f_2 must be replaced by F_1 and f_1, the first focal point and length. By considering plane wave fronts which are perpendicular to other (i.e., secondary) axes, the properties of focal planes may be investigated.

of its curvature and the refractive index of the material in which it travels. An examination of Figs. 4, 5, 6, and 7 then reveals that the changes in convergence $\{\pm 2n/r$, or $\pm(n-n')/r\}$ are the same in Fig. 6, where the initial convergence is zero, and in Fig. 7; whereas at a plane boundary (for which $r=\infty$) no change in convergence is to be expected or is actually produced. Further, from Fig. 8 it may be observed that, when the convergences of incident waves at a spherical surface change from the Fig. 7 values, then there are equal changes from the Fig. 7 values in the corresponding convergences of the reflected and refracted waves. Remembering that the convergences concerned are those *at the surface*, it is instructive to write this result in the form:

> Convergence of a reflected or refracted wave—the convergence of the reflected or refracted wave having as focus the center of curvature of the surface
>
> = convergence of corresponding incident wave—the convergence of the incident wave having as focus the center of curvature of the surface.

Or, expressed in symbols,

$$c' - c_N' = c - c_N.$$

This leads to $c' = c + (c_N' - c_N) = c + P$, say, which it is convenient to remember as

final convergence = initial convergence + power,

where power (or converging power) has a value which is most readily derived from Fig. 7 and may be recalled by means of Fig. 2. Inasmuch as converging power is the constant change $(c'-c)$ in convergence which occurs at the reflecting or refracting surface, it is conveniently defined as the convergence of the wave which results from the reflection or refraction of an incident plane wave ($c=0$), i.e., as the refractive index of the material in which the light finally travels divided by the second focal length. (Compare Figs. 6 and 7.) If refractive index is taken to be dimensionless, it follows that power has dimensions L^{-1} and is measured in such units as ft^{-1}, cm^{-1}, or $meter^{-1}$. The $meter^{-1}$ is usually adopted, and is called a *diopter* (abbreviation: D).

Now an image is real or virtual according as

the light converges or diverges after reflection or refraction, i.e., according as c' is positive or negative: moreover the image will be situated at the center of curvature of the reflected or refracted wave at a distance l' from the surface given by $|n'/c'|$. Thus the value of c' specifies the nature and position of the image.

To investigate the transverse linear magnification m, a secondary axis is drawn through N, Fig. 9. Then if a point object at O gives rise to an image at I, a similar object at O' will produce an image at I'; and, if OO' is small, OO' and II' may be regarded as perpendicular to the principal axis. Thus m will be the ratio of the distances of I and O, respectively, from N, and, moreover, the image will be inverted if I and O are on opposite sides of N. Now the equation $c-c_N=c'-c_N'$, takes the form $c(1-c_N/c)=c'(1-c_N'/c')$, or

$$c\{1\pm l/r\}=c'\{1\pm l'/r\},$$

and, if c and c' are of opposite sign, one of the terms in { } brackets must be positive and one negative, i.e., either the first is $(r\pm l)/r$ and the second $-(l'-r)/r$, or the first is $-(l-r)/r$ and the second $(r\pm l')/r$, the quantities in ( ) brackets being positive and representing possible values of NO and NI. In either case the image is on the opposite side of N to the object (Fig. 9), i.e., it is inverted with respect to the object. In like manner it may be shown that if c and c' have the same sign, i.e., the terms in { } brackets are

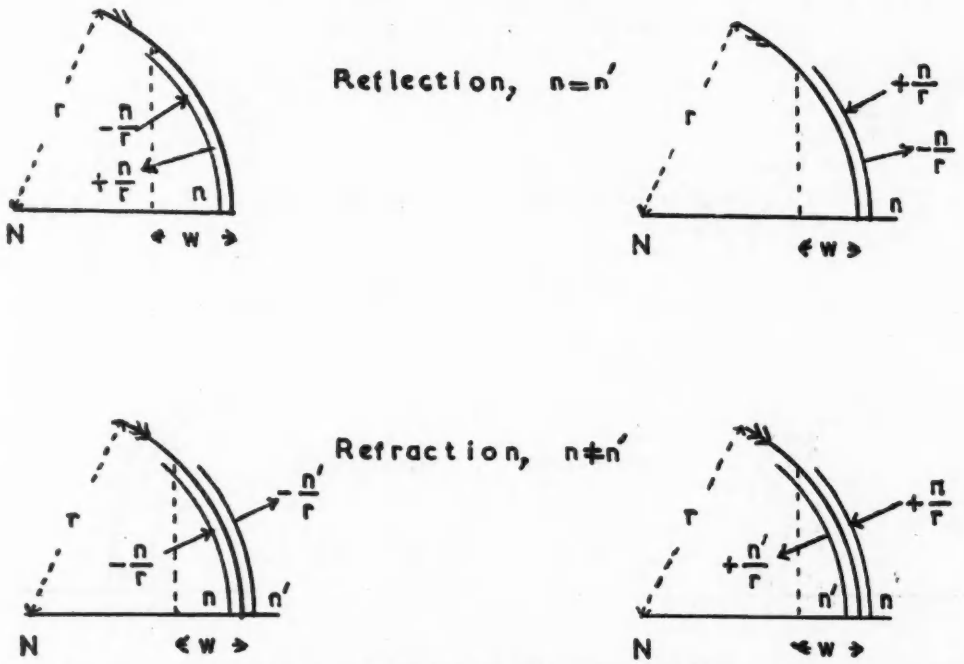


FIG. 7. Convergence at spherical surfaces of waves having nodal point as focus. Values of (final convergence—initial convergence) are

$[+n-(-n)]/r=2n/r$ $\qquad$ $[-n-(+n)]/r=-2n/r$

for reflection, and

$[-n'-(-n)]/r=[(n-n')]/r$ $\qquad$ $[+n'-(+n)]/r=[-(n-n')/r]$

for refraction.

Converging surface ($+ve$ power). $\qquad$ Diverging surface ($-ve$ power).

Note that these values (subsequently called the powers of the reflecting and refracting surfaces) may be written (±refractive index of material in which incident light travels ±refractive index of material in which reflected or refracted light travels)/r, the $+ve$ (or $-ve$) sign being used when the material is on the concave (or convex) side of the surface. This gives Fig. 2 an additional significance.

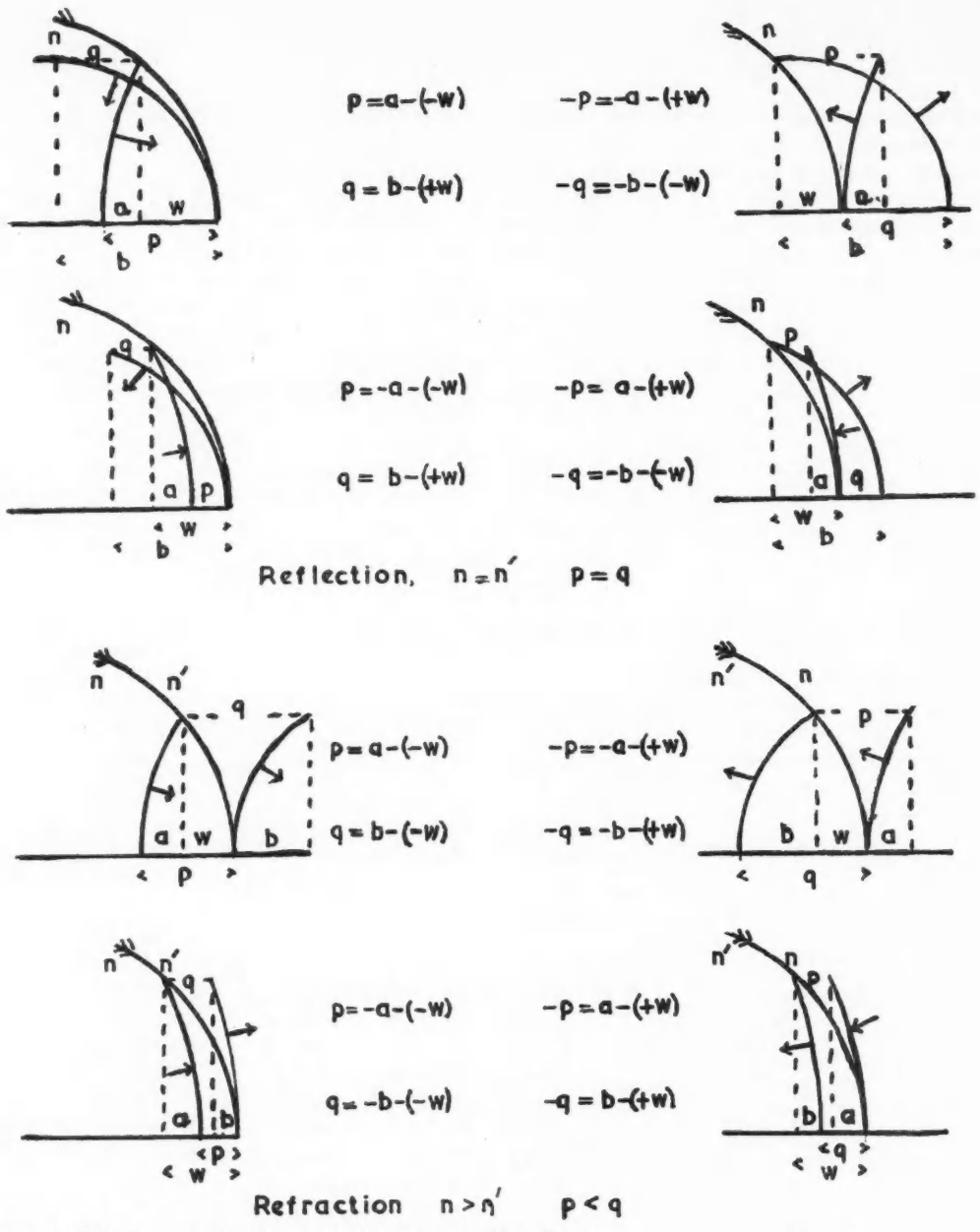


FIG. 8. Reflection and refraction of spherical waves at spherical surfaces.

Converging surface (+*ve* power). Diverging surface (−*ve* power).

Note (1). In terms of Fig. 3, a, b, and w are, respectively, the values of x, corresponding to a fixed value of y, for the incident and reflected (or refracted) waves and the surface. As the distances p and q are traversed in the same time, $q/v' = p/v$, or $n'q = np$. (2) If the sags are regarded as having the same signs as the curvatures to which they are proportional, then it may be observed that $|p|$ is the difference between the sags of the incident wave front considered and the incident wave front in the corresponding diagram of Fig. 7, and $|q|$ is the difference between the sags of the reflected or refracted wave front and the corresponding reflected or refracted wave front of Fig. 7, *due regard being had to the signs of the sags*. (3) The diagrams are arranged to emphasize similarity between the action of a system of +*ve* power on a wave front of +*ve* (or −*ve*) curvature and a system of −*ve* power on a wave front of −*ve* (or +*ve*) curvature. (4) If the wave fronts travel in opposite directions to those indicated in the diagrams, the equations for p and q (concerned now with the reflected or refracted and incident waves, respectively) may be obtained from those given above by multiplying throughout by −1. (5) If the wave fronts of Figs. 6 and 8 are regarded as advancing in the directions shown and also in the directions opposite to those shown, then Figs. 6–8 indicate various possible positions of conjugate foci.

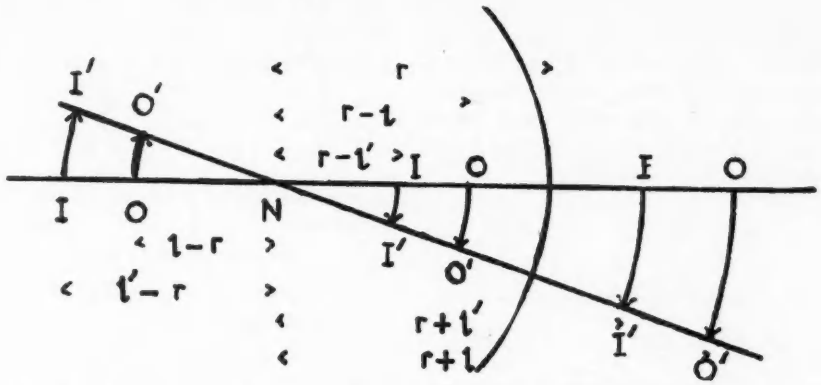


FIG. 9. Distances of objects and images from N in terms of their distances l and l', respectively, from the pole of the surface of radius of curvature r. $ON=O'N$ and $IN=I'N$.

either both positive or both negative, then the object and image are on the same side of N and erect with respect to each other. Thus a change in the sign of convergence on reflection or refraction (c/c' negative) indicates inversion of the image with respect to the object; and, moreover,

$$|c/c'| = |(r \pm l')/(r \pm l)| = m.$$

If we revert to Fig. 6 and the relation $c'=c+P$, we note that when $c=0$, $c'=P=\pm n'/f_2$; and, in addition, when $c'=0$, $c=-P=\mp n/f_1$, according as P is positive or negative. Thus $(f_2 \sim f_1)=(n' \sim n)/P$ and is zero for mirrors ($n=n'$), and r for refracting surfaces; moreover, as r is essentially positive,[1] the greater focal length is always on the concave side of the refracting surface. This conclusion directs attention to a close analogy, which merits emphasis, between a single refracting surface and a thin lens. Such a lens is represented in Fig. 10, in which P_1 and P_2 are the powers of two spherical refracting surfaces so near together that the convergence of the wave front emerging from the first surface does not change appreciably before it meets the second. It follows that $c_0=c+P_1$ and

$$c'=c_0+P_2=c+(P_1+P_2)=c+P, \text{ say.}$$

Moreover, as before, the transverse linear magnification $m=c/c'$, being the product $(c/c_0)(c_0/c')$ of the magnifications produced by the two refractions. Again, we may derive the values of f_1 and f_2 by putting $c'=0$ and $c=0$, respectively, and show that, as for a single refracting surface, $(f_2 \sim f_1)=(n' \sim n)/P=r$, say, where r is now to be interpreted as the distance from the lens of a point N situated on the same side of the lens as the longer focal length. (Figure 11.) Since the expression for r is equivalent either to $-n'/r=-n/r+P$ or to $n'/r=n/r+P$, it follows that N is, as before, a *nodal point*,[2] i.e., a point such that light originally diverging from or converging towards it continues to do so after refraction. Point N must moreover, by symmetry, be situated on the lens axis, i.e., a line joining the centers of curvature of the surfaces. Thus a positive (or negative) lens behaves like a positive

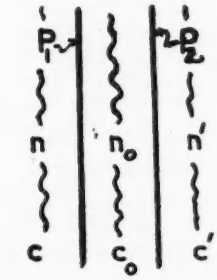


FIG. 10. Schematic diagram of thin lens.

[1] Signs are ascribed to convergences, and therefore powers, not to lengths.

[2] The existence of the nodal point at a distance $(f_2 \sim f_1)$ from the lens may also be inferred geometrically. If the image of a finite object is located by means of two rays, one associated with the first and one with the second focal point, then the line joining the tip of the object and its image will cut the axis at N. A comparison of results obtained from similar triangles and from the relation $c'=c+P$, then shows that $SN=(f_2 \sim f_1)$, where S is the lens center. If $n=n'$, $f_1=f_2$, $SN=0$, and the drawing of an undeviated ray through S is justified.

(or negative) refracting surface whose power is the sum of the powers of the refracting surfaces of the lens and whose radius of curvature is $(f_2 \sim f_1)$. This analogy, which will of course extend to the criterion for an inverted image, tends to be masked if it be assumed initially that $n = n'$ and N coincides with the lens center.

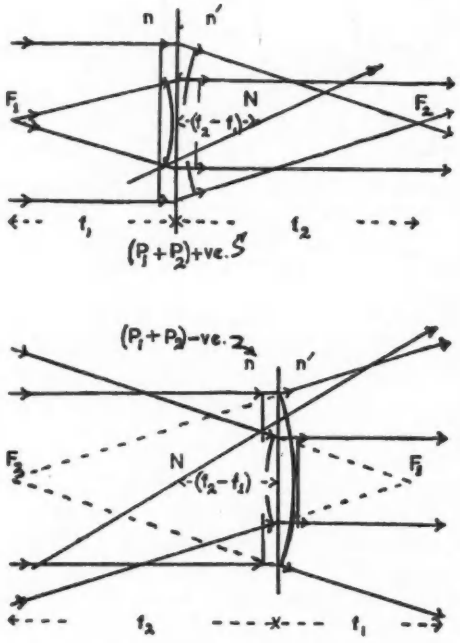


FIG. 11. Focal and nodal points of a thin lens. $(P_1+P_2) = P = \pm n'/f_2 = \mp n/f_1 = \pm(n'-n)/(f_2-f_1)$.

III. PROGRESSIVE CHANGES IN THE CONVERGENCE (OR CURVATURE) OF WAVE FRONTS DUE TO PROPAGATION IN A HOMOGENEOUS MEDIUM

Consider a wave front of convergence C proceeding in a medium of refractive index $\bar{n}$. The convergence C' of the wave front after it has advanced a distance X is, by definition,

$$C' = \pm\frac{\bar{n}}{A \mp X} = \pm\frac{1}{(A/\bar{n}) \mp (X/\bar{n})}$$

$$= \pm\frac{1}{\pm(1/C) \mp (X/\bar{n})} = \frac{C}{1 - C(X/\bar{n})},$$

according as the wave is converging or diverging. (Fig. 12.) Moreover, if the light is originally converging, the sign of C' will change when $X > A$. The formula may also be written

$$C = \frac{C'}{1 + C'(X/\bar{n})},$$

for the purpose of determining the convergence of the wave front in a previous position.

There follow three applications of this theorem.

Application 1. The derivation of the formula for longitudinal magnification.—Consider an object in the form of an arrow, tip A and tail B, placed *along* the axis of a reflecting or refracting surface and *pointing in the direction of the incident light* (Fig. 13). The image, $A'B'$, formed by paraxial rays may be determined in any particular case by locating the points conjugate to A and B, but the following analysis leads to two generalizations worthy of note.

At the surface the convergence (c_A) of the wave with center A is, having regard to sign as well as magnitude, less than that (c_B) of the wave whose center is B. This is true whether the light is diverging or converging, i.e., whether $OA < OB$ or $OA > OB$, where O is the pole of the surface. As reflection or refraction alters the convergences of both waves by the appropriate value of the power P, it follows that the convergence $c_{A'}$ of the wave whose center is A' is less than that $c_{B'}$ of the wave whose center is B'. Thus for a real image (formed by converging rays) $OA' > OB'$, and for a virtual image (formed by diverging rays) $OA' < OB'$. Hence *the image always points in the direction of propagation of the reflected or refracted light.*

Again, c_A is the value of the convergence of the wave of center B when it is at a distance AB from the surface, so that

$$c_B = c_A/(1 - c_A AB/n), \quad \text{or} \quad c_B - c_A = c_A c_B AB/n.$$

Moreover $c_{A'}$ becomes $c_{B'}$ at a distance $A'B'$ from the surface, so that

$$c_{B'} = c_{A'}/(1 - c_{A'} A'B'/n'),$$

or

$$c_{B'} - c_{A'} = c_{A'} c_{B'} A'B'/n'.$$

Now

$$c_{B'} = c_B + P \quad \text{and} \quad c_{A'} = c_A + P,$$

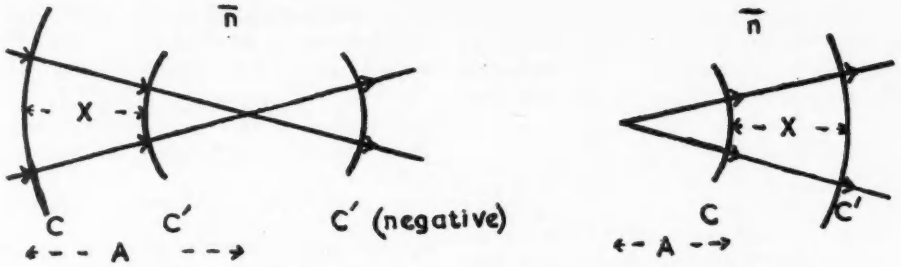


FIG. 12. Change in convergence due to progression of wave.
(a) Converging wave front. (b) Diverging wave front.

so

$$c_{B'}-c_{A'}=c_B-c_A \quad \text{or} \quad c_{A'}c_{B'}A'B'/n'=c_Ac_BAB/n,$$

and the longitudinal magnification

$$m_L=A'B'/AB=(c_A/c_{A'})(c_B/c_{B'})(n'/n)$$
$$=(n'/n)m_Am_B,$$

where m_A and m_B are the *transverse* linear magnifications appropriate to points A and B. If $n=n'$ and A is so near to B that $m_A=m_B$, this reduces to $m_L=m_A^2$, so that, for small objects and images in the same material, the longitudinal magnification is the square of the transverse magnification.

This analysis, being independent of the value of P, must also apply to a lens.

Application 2. The derivation of Newton's formula $c_{F_1}c_{F_2}=-P^2$.—For systems (mirrors, re-

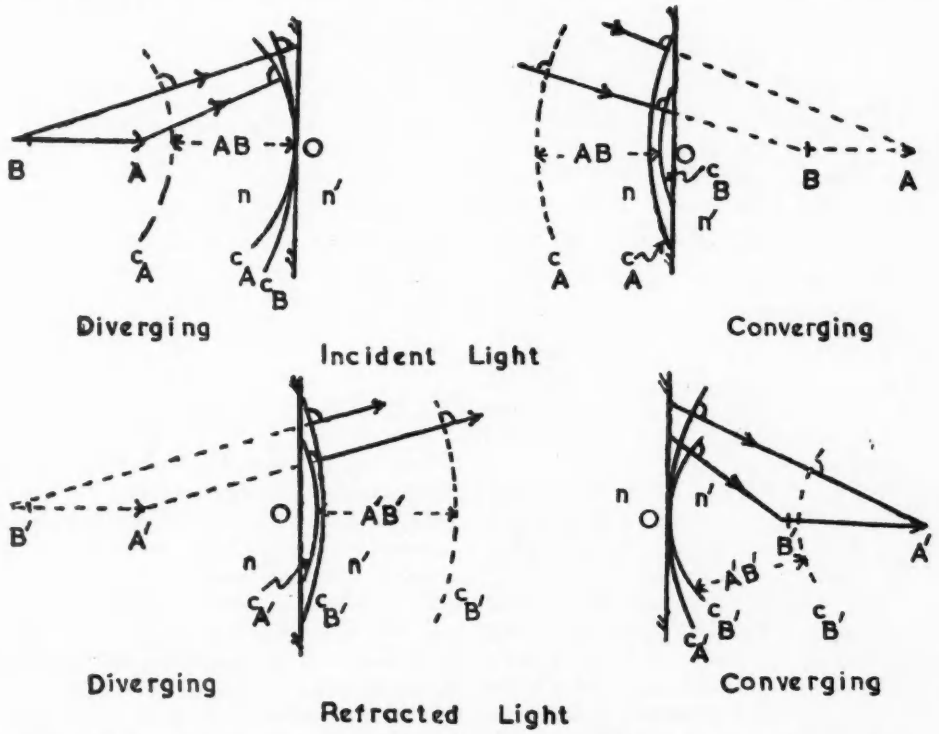


FIG. 13. Formation of the image of an axial object. For reflection, consider the two lower diagrams to be rotated 180° about the trace of the surface as axis.

fracting surfaces, and lenses) of positive power in which the incident light meets the first focal plane and then travels a distance f_1 to the system (see Fig. 11), the convergence at the first focal plane

$$c_{F_1}=\frac{c}{1+c(f_1/n)}=\frac{c}{1+c/P}=\frac{cP}{c+P}=\frac{c}{c'}P,$$

where c and c' are the convergences at the system of the initial and final wave fronts. The light subsequently travels a distance f_2 before it meets the second focal plane where the convergence is, in consequence,

$$c_{F_2}=\frac{c'}{1-c'(f_2/n')}=\frac{c'}{1-c'/P}=\frac{c'P}{P-c'}=-\frac{c'}{c}P.$$

For systems of negative power, c would become c_{F_1} if the light were to continue a distance f_1 in the medium of refractive index n, so

$$c_{F_1}=\frac{c}{1-c(f_1/n)}=\frac{c}{1-c/(-P)}=\frac{cP}{c+P}=\frac{c}{c'}P;$$

and c' is the convergence which a wave of convergence c_{F_2} would acquire in traveling a distance f_2 in a medium of refractive index n', so that

$$c_{F_2}=\frac{c'}{1+c'(f_2/n')}=\frac{c'}{1+c'/(-P)}=\frac{c'P}{P-c'}=-\frac{c'}{c}P.$$

Hence, whatever the sign of the power, the transverse linear magnification

$$m=c/c'=c_{F_1}/P=-P/c_{F_2}, \quad \text{and} \quad c_{F_1}c_{F_2}=-P^2.$$

It follows that c_{F_1} and c_{F_2} are of opposite sign. Thus, e.g., an object placed at a distance less than f_1 from a converging lens, and regarded as formed by a converging beam cutting the first focal plane, gives rise to a refracted beam which is diverging as it cuts the second focal plane. Again, as erect images are formed when c/c' is positive, erect images are also formed when c_{F_1}/P and $-P/c_{F_2}$ are positive.

Application 3. Refraction by separated systems. —(E.g., two separated refracting surfaces forming a "thick" lens, or two separated "thin" lenses). Let P_1 and P_2 (Fig. 14(a)) represent the effective powers of two systems of sensibly coincident refracting surfaces separated by a thickness, t, of material of refractive index n_0. A wave front of convergence c incident, in a material of refractive index n, on the first system acquires a convergence of $(c+P_1)$ after refraction: this becomes

$$(c+P_1)/[1-(c+P_1)t/n_0],$$

when the wave front meets the second system and

$$c'=(c+P_1)/[1-(c+P_1)t/n_0]+P_2 \qquad (1)$$

on entering the medium of refractive index n'. Equation (1) may be written

$$(c'-P_2)-(c'-P_2)(c+P_1)t/n_0=c+P_1,$$

or

$$\begin{aligned}c'-c=&(P_1+P_2-P_1P_2t/n_0)\\&-cP_2t/n_0+c'P_1t/n_0\\&+cc'(t/n_0)(P_1+P_2-P_1P_2t/n_0)/P,\end{aligned}$$

if

$$P_1+P_2-P_1P_2t/n_0=P. \qquad (2)$$

Hence,

$$\begin{aligned}c'(1-cP_2t/n_0P)-c(1+c'P_1t/n_0P)&\\=P\{1-cP_2t/n_0P+c'P_1t/n_0P&\\-cc'(t/n_0)^2P_1P_2)/P^2\},&\end{aligned}$$

or

$$c'/(1+c'P_1t/n_0P)=c/(1-cP_2t/n_0P)+P,$$

which it will be convenient to write

$$\bar{c}'=\bar{c}+P. \qquad (3)$$

But $\bar{c}$ and $\bar{c}'$ may be interpreted as the convergences which, in the absence of the refracting surfaces, the incident and (actual) refracted wave fronts would have at the planes which are indicated in Fig. 14(b) and defined by

$$(\alpha/n)=(t/n_0)(P_2/P)$$

and

$$(\beta/n')=(t/n_0)(P_1/P). \qquad (4)$$

These first and second principal planes intersect the axis at the principal points H_1+ and H_2+. However if P_2/P is negative, α is negative and $\bar{c}$ becomes $c/(1+cP_2t/n_0P)$ and represents the convergence of the incident wave front at a plane (H_1-) which is nearer to the source than the first refracting system. Similarily, if P_1/P is negative, β is negative, and $\bar{c}'$ becomes $c'/(1-c'P_1t/n_0P)$ and represents the convergence of the refracted wave front at a plane (H_2-) which is further from the source than the second refracting system. (Figure 14(b).)

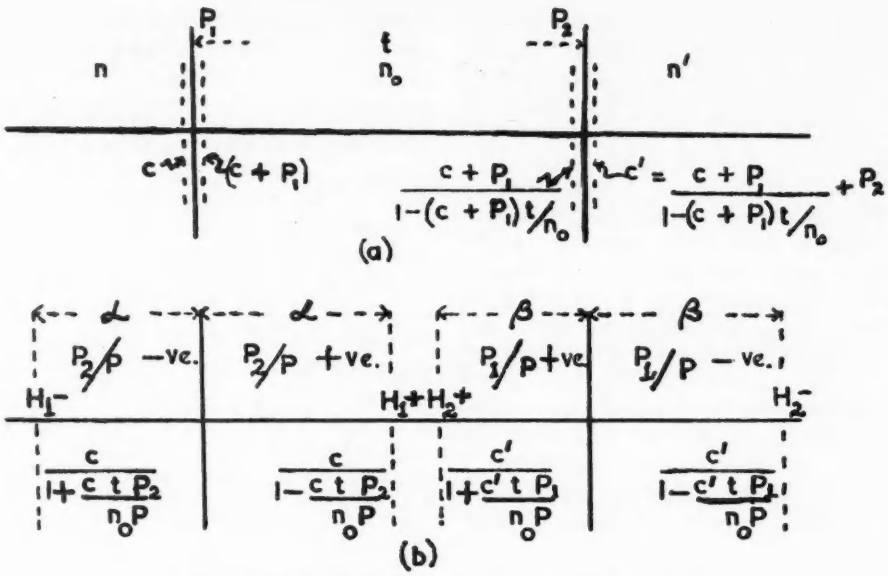


FIG. 14. Separated refracting systems and principal planes.

The transverse linear magnification produced by the two refractions,

M = magnification produced by the first refraction × magnification produced by the second refraction

$$= \left(\frac{c}{c+P_1}\right)\left(\frac{c+P_1}{1-(c+P_1)t/n_0}\cdot\frac{1}{c'}\right)$$

$$= \frac{c}{(c+P_1)+P_2\{1-(c+P_1)t/n_0\}}$$

(by Eq. (1))

$$= \frac{c}{(P_1+P_2-P_1P_2t/n_0)+c(1-P_2t/n_0)}$$

$$= \frac{1}{1+(P/c)(1-cP_2t/n_0P)}$$

$$= \frac{1}{1+P/\bar{c}} = \frac{\bar{c}}{\bar{c}'}. \quad (5)$$

It follows from Eqs. (3) and (5) that, as P must be finite, when $\bar{c}=\infty$, then $\bar{c}'=\infty$, and $M=1/(1+P/\bar{c})=+1$. Thus if a point on the first principal plane is the center of curvature of an incident wave, then a point at the same distance from the axis on the second principal plane is the center of curvature of the refracted wave. For this reason the principal planes are known as unit planes, and *any* incident paraxial ray cutting the first principal plane at a distance h from the axis, will, after refraction, come from, or appear to come from, a point on the second principal plane at the same distance h on the same side of the axis.

Reverting again to Eq. (3), we observe that, if f_1 and f_2 are the distances F_1H_1 and F_2H_2 of the principal foci F_1 and F_2 from the first and second principal points, respectively, then when $\bar{c}'=0$, $\bar{c}=-P=\mp n/f_1$, and when $\bar{c}=0$, $\bar{c}'=P=\pm n'/f_2$ according as P is $\pm$. Thus P has a numerical value $n/f_1=n'/f_2$ and a sign, which, taken together, disclose the positions of F_1 and F_2. Now as f_1 and f_2 are by convention, designated the focal lengths of the system, it seems fitting to call $P=P_1+P_2-P_1P_2t/n_0$, the power. Provided the convergences of refracted waves are specified with respect to the second principal plane, this is in accordance with the previous definition, and, moreover, P reduces to the value of power appropriate to a single refracting surface (i.e., P_1 or P_2 when P_2 or $P_1=0$) or to a thin lens (i.e., P_1+P_2 when $t=0$).

To complete the analogy with a thin lens suggested by Eqs. (3) and (5), it remains to remark first, that the previously given proof of Newton's relation may be applied without modification, and second, that if *two* nodal points are postulated such that a ray directed towards one emerges in a parallel direction collinear with the other (Fig. 15), then to satisfy the unit magnification property, these points must occur on the same sides of, and at equal distances from the principal planes. Further, the points must be on the axis, otherwise a ray directed towards one point and parallel to the axis would emerge parallel to the axis, and not in the direction of F_2. Thus the points must be as indicated in Fig. 15, where r may be calculated from Eq. (3) by putting $-n'/r=-n/r+P$ or $n'/r=n/r+P$ according to whether the points are nearer to or further from the source than the principal planes. Hence

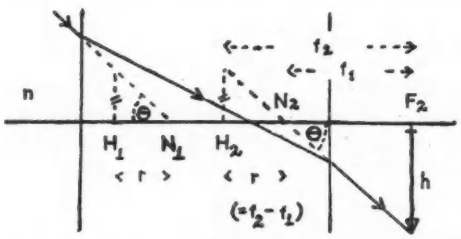


FIG. 15. Separated refracting systems and nodal points.

$r=\pm(n-n')/P=(f_1-f_2)$ or (f_2-f_1), and, as a negative value of r is meaningless, the nodal points are always displaced to the sides of the principal planes on which the focal length is longer.

Finally, by Fig. 15 it is possible to reconcile the definition of power adopted here with that contained in the Physical Society's "Report on the Teaching of Geometrical Optics." According to this Report the power ($\mathcal{P}$) of an optical element is the small angle subtended by an infinitely distant object multiplied by the refractive index of the material between the object and element and divided by the length of its image. Thus $\mathcal{P}=\theta n/h=n/f_1=P$.

IV. OPTICAL CALCULATIONS BY THE CONVERGENCE METHOD

There follow five numerical examples to illustrate the application of the formulae derived. It should be remembered that signs are attributed, not to lengths or directions, but to convergences, and therefore to powers, which are expressed in diopters unless otherwise stated.

(1) A mirror of power 4 D produces an image having a transverse linear magnification of 2. Find the two possible positions of the object.

With the previous notation, $m=c/c'=\pm 2$, according as the image is erect or inverted. If we substitute $c'=c+P$, we get $c/\pm 2=c+4$, or $c=-8$ D or $-(8/3)$ D, i.e., light diverges from a plane either $\frac{1}{8}$ or $\frac{3}{8}$ m from the mirror.

(2) A point source of light on the axis of a thin biconvex lens and 25 cm from its center, has faint and bright real images 4 cm and 100 cm, respectively, from the lens. Calculate the refractive index of the glass of the lens.

We use the notation of Fig. 16(a) and assume that powers are additive, i.e., that there is no *progressive* change in convergence within the lens. Then the power of the lens for the "normal" image is $P=(-1+n)/r_1+(n-1)/r_2$, and for the "reflected" image is $P'=(-1+n)/r_1+2n/r_2+2n/r_1+(n-1)/r_2$. From $c'=c+P$, $+1=-4+P$, and $+25=-4+P'$, so that $P/P'=(n-1)/(3n-1)=5/29$, whence $n=12/7$.

(3) Let us suppose the refracting surfaces in the eye illustrated in Fig. 16(b) to be very near together and 2 cm from the retina, and estimate the positions of the near and far points.

For the far point:

$$1.33/0.020=c_r+(1.33-1)/0.008+(-1.33+1.42)/0.010+(1.42-1.33)/0.006.$$

For the near point:

$$1.33/0.020=c_p+(1.33-1)/0.008+(-1.33+1.42)/0.008+(1.42-1.33)/0.005.$$

Hence $c_r=+1.25$ D, $c_p=-4$ D, and the eye can focus the limiting wave fronts indicated in Fig. 16(b), on which are shown the positions of the required points.

(4) Let A, B and C be three collinear points such that $AB=40$ cm and $BC=50$ cm. At A is situated an object 2 cm long and perpendicular to ABC; and at B and at C are situated lenses of power 4 D and -2 D, respectively, with their axes along ABC. Find the position, size, and nature of the image formed by the system.

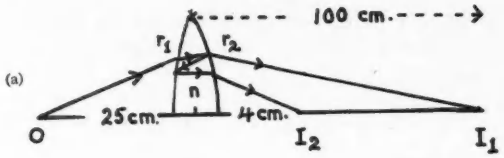
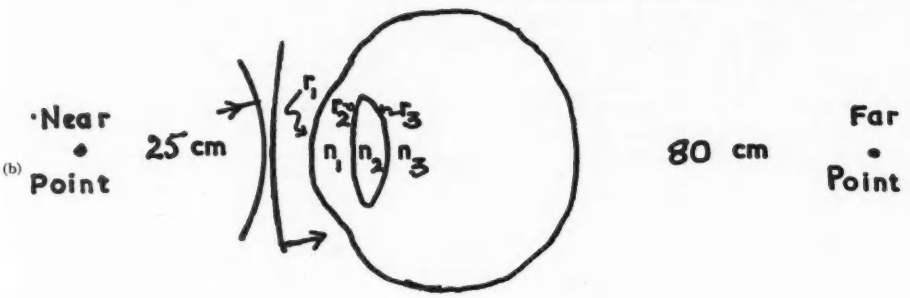
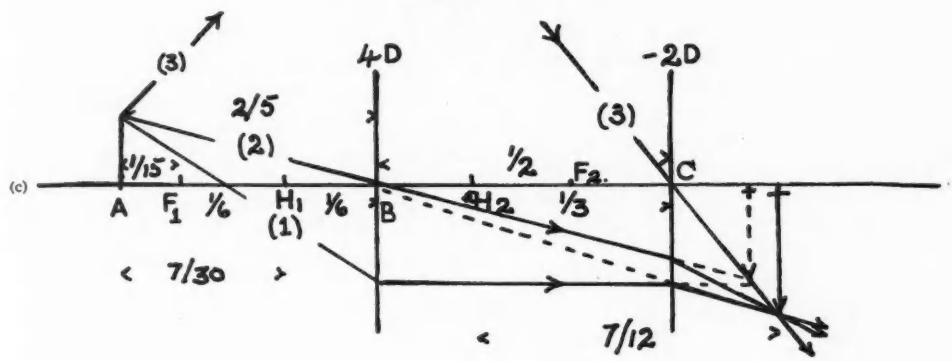
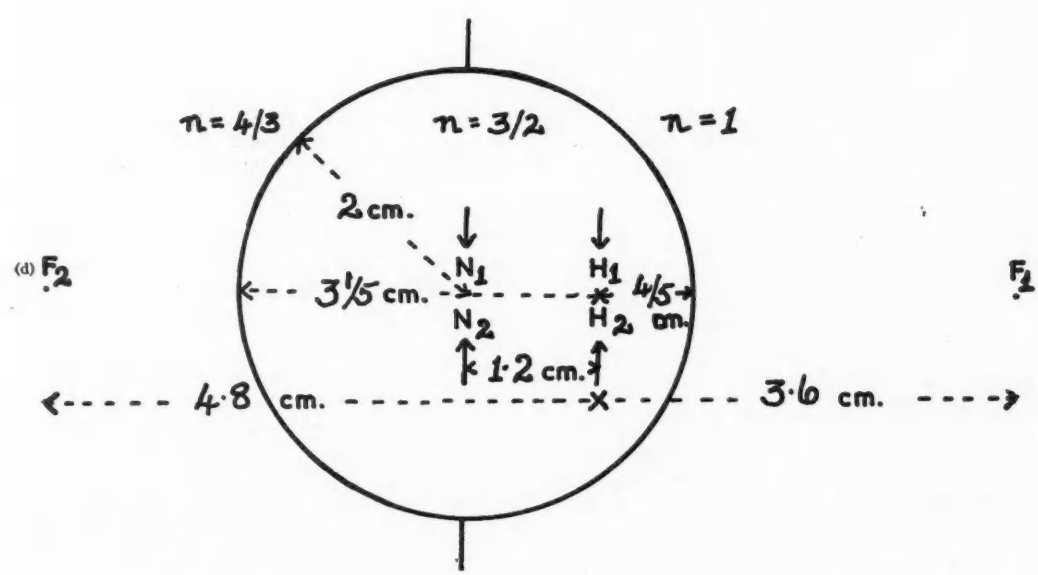


FIG. 16. (a) Problem 2. (b) Problem 3. $n_1=n_3=1.33$, $n_2=1.42$, $r_1=8$ mm. For far point, $r_2=10$ mm, $r_3=6$ mm; for near point, $r_2=8$ mm, $r_3=5$ mm. (c) Problem 4. Note that rays (1) and (2) locate the image due to the first refraction, and (1) and (3) that due to the second refraction. (d) Problem 5.







If we treat each lens separately, we have for refraction at the First lens, $c' = -1/0.4+4 = 1\frac{1}{2}$ D, i.e., light converges to a plane $(\frac{2}{3})$ m from B or $(\frac{1}{6})$ m from C; $m_1 = (-2\frac{1}{2}/1\frac{1}{2})$: and at the Second lens, $c' = +6-2 = 4$ D, i.e., light converges to a plane $(\frac{1}{4})$ m from C, and $m_2 = (6/4)$. Total magnification $= m_1 m_2 = -2\frac{1}{2}$. Therefore the image is real and situated 25 cm beyond C: it is inverted and $2 \times 2\frac{1}{2} = 5$ cm long.

If we treat the system as a "compound" one, we have $P = 4-2-(4)(-2)\frac{1}{2} = 6$ D; $\alpha = \frac{1}{2}(-2)/6 = -(\frac{1}{6})$ m; $\beta = \frac{1}{2}(4)/6 = (\frac{1}{3})$, so that the principal and focal planes are as indicated in Fig. 16(c). Further, as $\bar{c}' = -(30/7)+6 = (12/7)$ D, the light converges to a plane (7/12) m from H_2 or $(\frac{1}{4})$ m from C. Magnification $= -(30/7)/(12/7) = -2\frac{1}{2}$ as before.

Alternatively, Newton's formula gives $-15c_{F_2} = -6^2$, or $c_{F_2} = +12/5$ D. Thus light diverging from the object (1/15) m away from the first focal plane converges to a real focus (5/12) m beyond the second focal plane.

(5) A glass sphere, diameter 4 cm, whose material has a refractive index $\frac{3}{2}$, is surrounded on one side by air and on the other by a liquid whose refractive index is 4/3. Determine the positions of the cardinal points. Stops limit the rays to the paraxial region.[3]

$$P = \frac{\frac{3}{2}-1}{2} + \frac{\frac{3}{2}-(4/3)}{2} - \frac{1}{4}\frac{1}{12}\frac{4}{\frac{3}{2}} = \frac{10}{36}\ \text{cm}^{-1}.$$

Therefore

$$f_1 = \frac{1}{(10/36)} = 3.6\ \text{cm}; \quad f_2 = \frac{4/3}{(10/36)} = 4.8\ \text{cm};$$

and

$$(f_2 - f_1) = 1.2\ \text{cm}.$$

$$\alpha = \frac{4}{(\frac{3}{2})}\frac{1}{12}\frac{36}{10} - 1 = \frac{4}{5}\ \text{cm};$$

$$\beta = \frac{4}{(\frac{3}{2})}\frac{1}{4}\frac{36}{10}\frac{4}{3} = \frac{16}{5}\ \text{cm}.$$

The cardinal points are therefore situated in the positions indicated in Fig. 16(d), the nodal points, as might be expected, coinciding at the center of the sphere.

[3] *Cf.* C. J. Smith, Phil. Mag. **37**, 585 (1946).

Nothing is eternal save Heaven. Romes existed before Rome, and when Rome will be a waste there will be Romes after her. Seek out some city that is young. The secret is to make a city not to rest in it. When you have found one, drink in the illusion that she too is eternal.—THORNTON WILDER.

A Dynamic Model of Magnetic Resonance Phenomena*

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(Received April 22, 1953)

A dynamic model to illustrate atomic and nuclear magnetic phenomena is described. An air-driven spinning magnetic dipole placed in a magnetic field exhibits a Larmor precession whose dependence on magnetic and gravitational parameters can be demonstrated. An ac magnetic field of variable frequency permits a visual demonstration of magnetic resonance.

Data obtained from the precession experiments can be used for a precise determination of the earth's magnetic field.

THE concept of a spinning magnetic dipole in an external magnetic field and its interaction with an alternating magnetic field is commonplace in both atomic and nuclear physics. It is basic to the understanding of such phenomena as Russell-Saunders coupling, Zeeman splitting of spectral lines, molecular-beam magnetic resonance, nuclear magnetic absorption, and nuclear magnetic induction. The dynamics involved in these phenomena are capable of being demonstrated on a macroscopic scale. The purpose of this article is to describe a device which does this, and to indicate how it has been used as a laboratory teaching aid.

THEORY

A magnetic dipole of magnetic moment **M** and angular momentum **P** will precess when an external torque is applied to it. The applied torque in a large-scale model will be the vector sum of a magnetic torque and a gravitational torque. The relation is

$$d\mathbf{P}/dt=\mathbf{M}\times\mathbf{H}+\mathbf{r}\times\mathbf{w}, \tag{1}$$

where **H** is the external magnetic field, **r** is the position vector of the center of mass relative to the point of support, and **w** is the weight of the entire spinning top. One possible orientation is represented in Fig. 1. The field **H** is produced by a pair of Helmholtz coils and can therefore be reversed in direction; the moment **M** can be reversed by reversing the orientation of the dipole (a cylindrical Alnico magnet) in the top.

If the frequency of precession ω_L is much less than the frequency of spin,

$$d\mathbf{P}/dt=\boldsymbol{\omega}_L\times\mathbf{P}. \tag{2}$$

Combining with Eq. (1) and noting that the angle θ involved in all the cross products is the same,

$$\omega_L=(rw\pm MH)/P. \tag{3}$$

The positive sign is used when the gravitational and magnetic torques are in the same direction. For opposing torques (which can be achieved by reversing either **M** or **H**), the precessional frequency can be reduced to zero, and even changed in direction, by varying **H**. The condition for no precession is given by

$$H=rw/M, \tag{4}$$

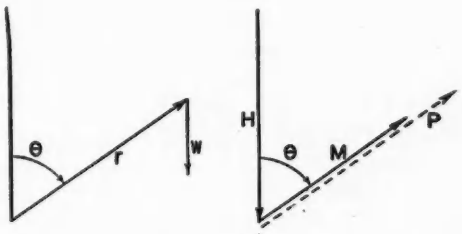

FIG. 1. Possible relative orientation of quantities involved in the motion of a spinning magnetic dipole.

and obviously does not depend on the magnitude of the angular momentum, P. The latter can be changed by varying the frequency of spin; in this respect, the laboratory model has an advantage over its atomic and nuclear counterparts.

It should be pointed out that the above analysis assumes that M remains constant. That is, it is not demagnetized by H, which must therefore be less than the coercive field H_c for the magnetic material being used. This condition can readily be achieved, as will be indicated below.

* The apparatus described in this paper was developed in connection with a research project supported by the Research Corporation.

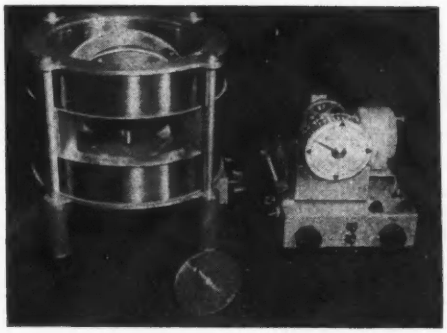

FIG. 2. Experimental apparatus. The Helmholtz-coil combination is at the left; the ac generator at the right. The magnetic top is in the foreground.

CONSTRUCTION

The design of the magnetic top and the Helmholtz coils is determined by a number of interrelated factors. If one takes $r=3$ cm, $w=60$ g or 6×10^4 dynes, and $M=1000$ ergs/oersted, the order of magnitude of H can be determined from Eq. (4). The result is 180 oersteds. The coercive field H_c for Alnico 5 is about 550 oersteds; it can be expected, therefore, that the magnetic moment M will remain very nearly constant in a magnetic field of the magnitude indicated. This has been verified experimentally.

Given the maximum magnetic field strength one wishes to obtain and the arbitrary size one desires for the coils, the size of the copper wire to be used is determined uniquely by the voltage of the available source of dc power (assuming no limit to the current that may be drawn through the source). This follows from the fact that H is proportional to the ampere-turns product, NI; the resistance of the coils, on the other hand, is proportional to N. This, combined with the volume of the windings will specify the proper wire size for a given battery voltage. The unit we have constructed was designed to give a field strength of about 200 oersteds, using a 120-volt battery for two coils in series; each coil having an inner diameter of 8 inches and an outer diameter of 10 inches. The wire used was No. 19, Formvar-covered. Each coil has 1485 turns. The assembled unit is shown in Fig. 2. The second set of coils at right angles to the main coils will be described below.

The construction of the top is indicated in Fig. 3. To keep its weight small [the importance of which is indicated by Eq. (4)], the main housing is constructed of aluminum, and the central circular disk of Plexiglas. The aluminum housing consists of two concentric shells A and B constructed as shown. There is a centering disk at the bottom end of shell B. Most of the material of this disk was removed by filing, leaving three pins to provide support between the two shells. The bearing surface of the top is a 3/16-inch steel ball bearing, press-fitted into the aluminum housing. A hole bored through the steel ball allows compressed air to enter the top from tube C on which the top is supported. The air passes up past the centering pins and through the housing, moves out radially through the Plexiglas and finally leaves the top tangentially. The resulting couple keeps the top in motion. The effective air pressure is adjustable by a valve in the air line. The angular momentum of the top can then be varied by changing the pressure. The tube C is supported in the Helmholtz-coil frame by means of a rubber stopper through

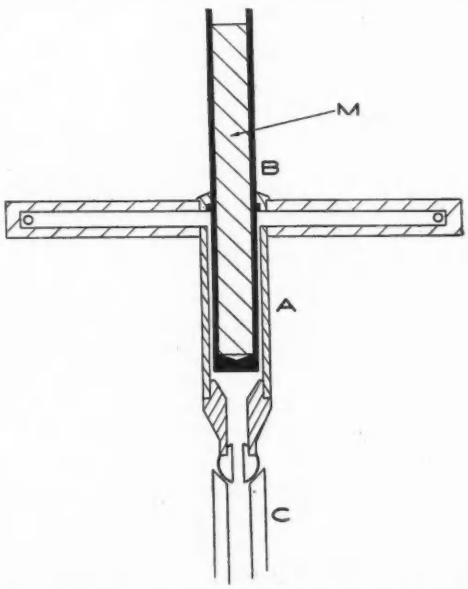

FIG. 3. Construction of the spinning dipole. The radial channels shown in the central disk are two $\frac{1}{8}$-inch holes drilled through the disk. The centering pins at the bottom of shell B have been omitted from the diagram for the sake of clarity.

which it passes. It was found that if the tube was rigidly attached to the frame, the turbulence of the air prevented the top from precessing smoothly. The bearing seat on the upper end of the tube C is conical in shape. In starting the top, its upper end is also fixed; that is, the top spins about a fixed axis. When the top has attained suitable angular velocity, the upper point of support is raised, and the top begins to precess about a vertical axis.

USE AS LABORATORY EQUIPMENT

The magnetic top is useful for a number of experiments in an advanced undergraduate laboratory. It is possible to test Eq. (3) experimentally by variation of some of the parameters on which it depends. It is possible also to obtain a precise value for the product MH. If H is then determined independently, the magnetic moment M can be calculated. Then, by measurement of the period of oscillation of the dipole in the earth's magnetic field, the value of the latter can be obtained to an accuracy which exceeds by an order of magnitude the results of the usual magnetometer method.

In testing Eq. (3) experimentally, the data are taken in such a way that r can be calculated from the results. A constancy in the value of r for a range of values of the other parameters thus constitutes an experimental check of the equation. In taking these data, ω_L is measured by a stopwatch (the periods of precession can be varied from about two seconds to infinity). For determining the angular momentum P, the moment of inertia of the top is measured by the torsion pendulum method. An accurately-milled brass cylinder of known dimensions is used as the reference standard. A hole drilled in the base of the cylinder is of the same diameter as the stem of the top. The period of oscillation with the top and without it gives directly the moment of inertia of the top. The angular velocity ω of the precessing top is measured by a stroboscope. A unit with a variable-speed mechanical switch was found to give accurate results.

The data for Eq. (3) were taken for a range of values of current in the Helmholtz coils from zero to two amperes. For zero current, only the gravitational torque is acting. A possible effect of the vertical component of the earth's magnetic field was eliminated by reversing the orientation of the dipole for half of all the readings. Subsequent calculations using the experimentally determined values for r, M, and w indicated that the magnetic torque for zero current was about 0.3 percent of the gravitational torque. For currents different from zero, the data were taken in pairs, with the current (and therefore H) reversed for the two cases. By combining the two results, the product MH can be eliminated, and r determined directly. The center of gravity of the top is thus determined by a measurement of a weight, a moment of inertia, and two frequencies.

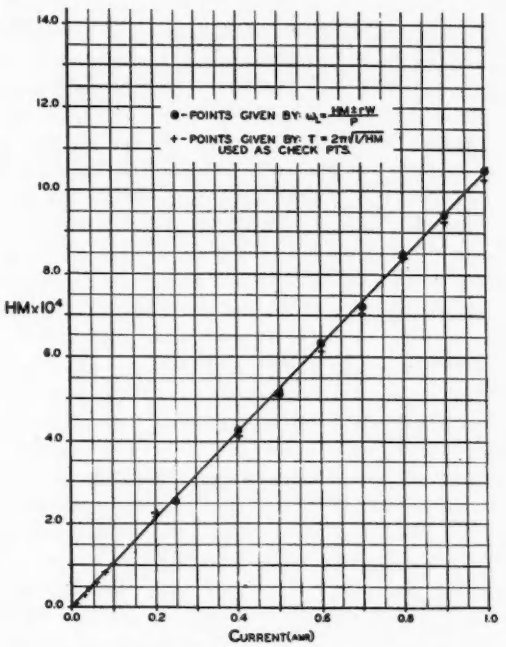

FIG. 4. Plot of MH *vs* current in the field coils. The curve is drawn through the points marked with circles. The points marked with crosses are comparison points.

The calculated result for r, using a range of values of P and ω_L was $r=3.266\pm0.005$ cm. No reading deviated from the mean by more than 0.05 cm. The results indicated that the buoyant effect of the air stream is negligible, since this would have effectively reduced w and therefore changed r for different air pressures.

Once r has been determined the product MH can be obtained as a function of the current I

in the coils. The apparatus can also be laid on its side, so that H is horizontal. The dipole, suspended freely in this field, oscillates with a period given by

$$T=2\pi(I_0/MH)^{\frac{1}{2}}, \tag{5}$$

where I_0 is the moment of inertia about an axis through the supporting string. The two sets of data are summarized in Fig. 4. For high current values, the results using Eq. (5) appear to be somewhat lower than those from Eq. (3). The reason for this discrepancy has not been determined. The oscillation frequencies are relatively high (about 10 cycles/sec) and are measured using the stroboscope. The motion is in this case free and damped. The damping tends to increase the period, and therefore effectively decrease MH. Also, for Eq. (3) the motion is isochronous so far as the orientation angle θ is concerned; the motion of Eq. (5) is not isochronous. This effect again increases the period.

It is not possible to determine M (or H) separately from Eqs. (1) to (5). This is analogous to the situation in the measurement of nuclear magnetic moments (where *ratios* of magnetic moments can be determined precisely in a magnetic field) and emphasizes the importance of measuring the proton magnetic moment either absolutely,[1] or in terms of the Bohr magneton.[2] However, if one measures H independently, M can be computed. The use of a proton resonance probe for the determination of magnetic field strengths has become quite general, and can readily be adapted to the advanced undergraduate physics laboratory, where its high precision makes it an extremely useful experimental tool. There are, of course, alternative methods for determining H.

We determined H for a particular value of the current in the Helmholtz coils in the following manner. A "flip" coil with N turns of wire and of cross-sectional area A was placed in series with a galvanometer and a resistance box, so that the total resistance is R. The coil was flipped through 180° by a torsional spring action and, at the end of the 180° rotation, the galvanometer circuit opened automatically by having the coil act on a microswitch. Thus the galvanometer was approximately ballistic. The galvanometer deflection d is given by

$$d=k(NAH_p)/R, \tag{6}$$

where k is a proportionality constant characteristic of the galvanometer. The flip coil was placed near the proton resonance probe, and while the resonance signal remained on the oscilloscope, a flip-coil measurement was taken with the galvanometer. From the frequency of the rf generator and the known gyromagnetic ratio of the proton, H_p can be determined. The flip coil was then placed in the magnetic field H, and the galvanometer deflection d measured. Because the magnetic field H_p is greater than H by a factor of about thirty-five, it was necessary to use a different coil, having a cross-sectional area A' and N' turns. The two coils were so made that their moments of inertia were nearly equal, so that the time involved in "flipping" through 180° was nearly identical in the two cases. The galvanometer was therefore about equally ballistic for the two measurements. Finally, the resistance R was adjusted until the deflections were about equal, so that any dependence of the proportionality constant k upon the deflection d could be neglected. From Eq. (6) it follows that

$$(NAH_p)/Rd=(N'A'H)/(R'd'), \tag{7}$$

from which H can be calculated.

A representative set of results follows: $HM=10.55\times10^4$ ergs for $I=1.00$ amp; $H_p=7046$ oersteds at a generator frequency of 2.9992×10^7 cycles/sec; $d=15.80\pm0.01$ cm; $d'=16.68\pm0.02$ cm at $I=1.00$ amp; $H=103.7$ oersted; $M=1.011\times10^3$ ergs/oersted. Quantities H and M have a precision of about 1 percent. The largest single source of error is an uncertainty in the resistance of the galvanometer. In the above experiment, this was measured by the half-deflection method. More precise methods are available to make this measurement.

The magnetic dipole of known M can be used to determine directly the horizontal component of the earth's magnetic field by use of Eq. (5). A representative result for Northfield, Minnesota, was found to be $H_e=0.164$ oersted. The

[1] Thomas, Driscoll, and Hipple, Phys. Rev. **75**, 902 (1949).

[2] J. H. Gardner and E. M. Purcell, Phys. Rev. **76**, 1262 (1949).

precision of H_e is approximately that of M. With greater care, particularly in the ballistic galvanometer measurements, an improved precision is readily attainable. The result above may be compared with the usual magnetometer method for measuring H_e. In the latter method, the experimental error usually exceeds 5 percent, even when considerable care is exercised in taking the data.

MAGNETIC RESONANCE

The interaction of the precessing magnetic dipole with an ac magnetic field whose frequency of oscillation approximately equals the frequency of precession is known as magnetic resonance. The ac field is at right angles to the dc magnetic field. The dynamics of interaction have been summarized in review articles by Pake,[3] and will not be reproduced here. Because the spinning magnetic top is an isolated dipole, the "spin-spin" and the "spin-lattice" interactions are absent, and the equations of motion are thereby simplified.

One particular characteristic of the magnetic resonance which is directly demonstrable with the macroscopic model is the phase relationship which must exist between the precessing dipole and the oscillatory magnetic field to give optimal response. This relationship will be derived. For this purpose, it is convenient to rewrite Eq. (1) in the form

$$d\mathbf{P}/dt=\mathbf{M}\times\mathbf{H}_0, \tag{8}$$

where $\mathbf{H}_0$ is the equivalent magnetic field strength which would produce the same magnetic torque as the combined magnetic and gravitational torques. We now add to $\mathbf{H}_0$ the ac field given by

$$H_x=H_1\cos\omega t. \tag{9a}$$

The other two components of the magnetic field are

$$H_y=0, \tag{9b}$$

$$H_z=H_0. \tag{9c}$$

[3] G. E. Pake, Am. J. Phys. 18, 438, 473 (1950). See particularly sections C and D of the first paper.

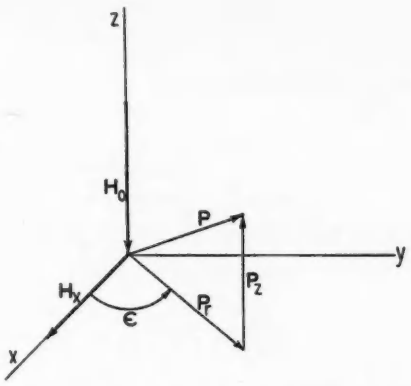

FIG. 5. Phase relationship between the ac field and the precessing dipole. See text.

The relative orientations of the dipole and the fields are shown in Fig. 5.

Expanding Eq. (8) into its component equations, we find

$$\frac{dP_x}{dt}=M_yH_0, \tag{10a}$$

$$\frac{dP_y}{dt}=H_1M_z\cos\omega t-M_xH_0, \tag{10b}$$

$$\frac{dP_z}{dt}=-H_1M_y\cos\omega t. \tag{10c}$$

From Eqs. (10a) and (10c) we obtain

$$\frac{dP_z}{dt}=-\frac{H_1}{H_0}\frac{dP_x}{dt}\cos\omega t. \tag{11}$$

If we now apply the condition of resonance, namely, that the frequency of precession equals the frequency of the applied oscillatory field,

$$P_x=P_r\cos(\omega t+\epsilon), \tag{12}$$

where P_r is the component of $\mathbf{P}$ in the xy-plane. This component P_r varies with time as the dipole absorbs energy from or radiates energy to the ac field. The phase angle ϵ may be visualized as the angle between P_r and a *rotating* magnetic field H_r of constant magnitude, whose projection on the x-axis is the actual field H_x. It follows

from Eqs. (11) and (12) that

$$\frac{dP_z}{dt}=\frac{H_1}{H_0}\left[\omega P_r \cos\omega t \sin(\omega t+\epsilon) - \frac{dP_r}{dt}\cos\omega t \cos(\omega t+\epsilon)\right]. \quad (13)$$

If $H_1 \ll H_0$, the orientation of the dipole changes only slightly per precession cycle. That is,

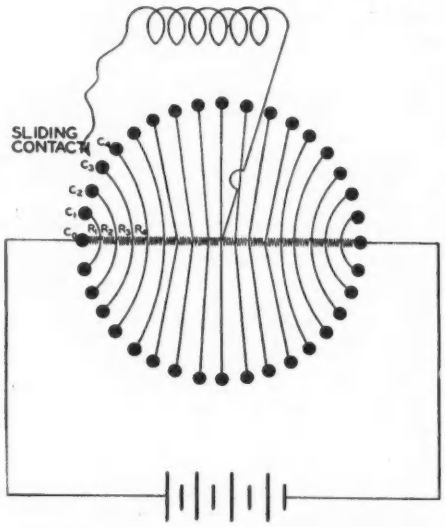

FIG. 6. Diagrammatic representation of the sinusoidal potentiometer used as an ac generator.

$dP_r/dt \ll \omega P_r$. With this approximation we average Eq. (13) over one cycle. The result is

$$\left\langle \frac{dP_z}{dt} \right\rangle_{Av} = \frac{H_1}{H_0}\langle \omega P_r \cos\omega t \sin(\omega t+\epsilon)\rangle_{Av}, \quad (14)$$

which can readily be shown to be

$$\left\langle \frac{dP_z}{dt} \right\rangle_{Av} = \frac{H_1}{2H_0}\omega P_r \sin\epsilon. \quad (15)$$

The potential energy of the dipole in the field H_0 equals $\mathbf{M}\cdot\mathbf{H}_0$. It follows that the average rate at which the dipole absorbs or radiates energy is proportional to $\langle dP_z/dt\rangle_{Av}$ (assuming that $\mathbf{M}$ and $\mathbf{P}$ are parallel). For the configuration of Fig. 5, energy is being absorbed if $\langle dP_z/dt\rangle_{Av}$ is positive. It follows from Eq. (15) that no power is absorbed from the field (or radiated to the field) if $\epsilon=0$ or π. Maximum absorption of power occurs at $\pi/2$ and maximum radiation at $3\pi/2$. These results can readily be checked qualitatively. If $\epsilon=\pi/2$, the ac magnetic field will be a maximum in the positive x direction when P_r is instantaneously along the positive y direction. Application of the cross-product rule will show that the resulting torque tends to erect the dipole in the positive z direction.

In our equipment, the ac coils had an average diameter of about 6.25 inches. Each coil had 4500 turns of No. 32 Formvar-covered wire. The period of precession of the dipole was of the order of seconds. The applied field must therefore have a frequency of less than one cycle per second. (This is in contrast to the rf frequencies of tens, or thousands, of megacycles per second for nuclear resonances or electron resonances, respectively, for the magnetic field strengths used in those instances.) For the low frequencies needed in the macroscopic model, the simplest ac generator is a converter, based on the use of a sinusoidal potentiometer. The circuit is shown in Fig. 6. The rotating contact is motor-driven at constant speed. The values of the resistors are so chosen that the resulting potential difference across the coils is a step function which approximates a sine curve. For example, the contact points C_1, C_2, C_3, etc., are spaced at ten-degree intervals. The resistance R_1 is proportional to sin 10°; R_1+R_2 is proportional to sin 20°, etc. The frequency of the resulting ac voltage can be varied by changing the speed of the motor.

The oscillating field can be "tuned in" to the precessing dipole by varying H, by changing the frequency of spin (i.e., changing the air pressure applied to the spinning top) or by changing the frequency of the field. Phase adjustment is controlled manually by momentarily opening a toggle switch in the line that supplies power to the motor. We have considered methods of automatic control, but have not tried them out. The manual phase adjustments are somewhat difficult to control; they can be done accurately enough, however, to check qualitatively the phase dependence indicated by Eq. (15).

It should be pointed out that the quantitative

measurements described earlier in this paper do not require an oscillating magnetic field. This is in contrast with the atomic and nuclear measurements which always require resonance techniques. In the macroscopic model, one needs no resonant bridge circuits and the associated detecting devices. Instead, one determines the precessional frequency by direct measurement and in this way determines directly the quantities which in the submicroscopic cases can be measured only by means of associated phenomena.

We are using the equipment for demonstration purposes in advanced undergraduate courses. Primarily, however, it is used for laboratory purposes at the senior level. As an experiment, it approximates an actual research situation in that the student must learn many techniques and make many measurements in order to arrive at a single experimental result. For example, he determines moments of inertia (by the torsion pendulum method), magnetic flux (with the ballistic galvanometer), current (with the potentiometer), resistance, frequency, and center of mass. The experiment takes several weeks to complete; but it does represent experimentation with a purpose.

The author is indebted to Mr. Russell Ferlen, who constructed the equipment, and to two senior students, Hollis L. Caswell and William W. Simmons, who took the experimental data.

Professor Robert S. Shankland sent us this photograph of the plaque commemorating the Michelson-Morley experiment and the centenary of Michelson's birth. The plaque was recently installed on the line joining the campuses of the Case Institute of Technology and Western Reserve University. Very appropriately it was dedicated on December 19, 1952, that day being the 100th anniversary of Michelson's birth, at a special meeting of the Cleveland Physics Society.

The inscription on the plaque reads: "Near this spot, in July 1887, Dr. Albert A. Michelson of Case and Dr. Edward W. Morley of Western Reserve University conducted the world-famous Michelson-Morley experiment, one of the outstanding scientific achievements of the 19th century, and a cornerstone of modern physics. In commemoration, this tablet has been set in stone by both colleges on December 19, 1952, the 100th anniversary of Dr. Michelson's birth."

Plaque commemorating Michelson-Morley experiment.

Parameters Characterizing the Strength of a Shockwave

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(Received April 8, 1953)

Inspection of the equations of compressible hydrodynamics in one dimension shows that the steady state may possess a discontinuity in the flow variables u, p, ρ. From this there arises the possibility of constructing a dimensionless parameter characterizing shock strengths. Its relations with other shock parameters are discussed.

I. INTRODUCTION

THE following remarks are concerned with normal shocks only, that is, with shocks which are intersected normally by the fluid velocity. A very weak shock approaches a sound wave in behavior; pressure and density in front of and behind it differ but infinitesimally; and its speed, as it penetrates the gas at rest, approaches the speed of sound. On the other hand, a strong shock represents a finite discontinuity of the flow variables, for example, of pressure and density, and it can move with a speed several times as great as the speed of sound of the gas into which it proceeds.

Several parameters which are intended to characterize numerically the strength of a shock have been suggested,[1] and it seems that few investigators in this field can agree on which parameter to choose. Here at Michigan we have been using the pressure ratio, namely the quotient of the pressure in front[2] of the shock and the pressure in back. This parameter, generally denoted by ξ, is between one for sound waves and zero for infinitely strong shocks and it is unsatisfactory in that a strong shock is characterized by a quite small ξ, when the very violence of the phenomenon would call for a large number. In view of this, ξ^{-1} denoted by y, might be preferable. The ratio of the speed of the shock to the sound speed of the gas in front has also been used and has been given the unfortunate name of "Shock Mach number." If the shock is transformed to rest, then this shock Mach number becomes the Mach number of the gas in front. Thus this parameter seems to prefer the flow on one side of the shock over that on the other.

The following simple considerations are intended to throw light on this question by going back to the foundations of the entire phenomenon, namely to the three shock conditions. A dimensionless shock parameter will be seen to arise which has the same form whether it is written in terms of the flow variables on one side of the shock or the other. Further it will be shown what the relations are between this shock parameter and others discussed in the literature.

II. THE SHOCK CONDITIONS

First let us recall the usual equations of compressible hydrodynamics. Restricting ourselves to one-dimensional phenomena we have the continuity equation as an expression of the mass balance,

$$(\partial\rho/\partial t)+(\partial\rho u/\partial x)=0; \tag{1}$$

secondly, the equations formulating the momentum balance,

$$(\partial\rho u/\partial t)+(\partial\rho u^2/\partial x)+(\partial p/\partial x)=0, \tag{2}$$

which is a ready consequence of Euler's equation together with Eq. (1); and finally the energy balance,

$$(\partial/\partial t)[\rho(E+\tfrac{1}{2}u^2)]+(\partial/\partial x)[\rho u(i+\tfrac{1}{2}u^2)]=0. \tag{3}$$

In these equations ρ, p, and u denote the density, the pressure, and the fluid velocity, respectively; furthermore $E=c_vT$ is the internal energy and i the enthalpy, is given by the equation $i=E+p/\rho=c_pT$.

Now let us search for such solutions, one might say such trivial solutions, which are independent of the time. Then all those expressions whose space derivatives appear above have to be con-

[1] See the extensive presentation in R. Courant and K. O. Friedrichs, *Supersonic Flow and Shock Waves* (Interscience Publishers, Inc., New York, 1948), esp. pp. 154 to 156.

[2] Since a shock is always intersected by fluid one can distinguish a "front" and a "back." The front is that side of the discontinuity which takes in the fluid, the back the side which ejects it.

stant; we shall denote the constants arising in this way from Eqs. (1), (2), and (3) by C_1, C_2, and C_3, respectively. One has

$$\rho u = C_1, \tag{4}$$

$$\rho u^2 + p = C_2, \tag{5}$$

and

$$\rho u(\tfrac{1}{2}u^2 + i) = C_3. \tag{6}$$

The physical meaning of C_1, C_2, and C_3 is obvious from these formulas; they are clearly the current densities of mass, momentum, and energy, respectively. Remembering that $i = (c_p/R)p/\rho$, one has, therefore, three quations which make it possible to express the field variables u, p, and ρ in terms of C_1, C_2, and C_3. Thus it appears that if one specializes the hydrodynamical equations to a flow which is both one-dimensional and steady, the only solutions are constant values for u, p, and ρ. The same situation arises for the Maxwell equations under similar assumptions. But the case of hydrodynamics is nevertheless different since the equations for u, p, and ρ are nonlinear and allow for several constant solutions. Upon elimination of ρ by means of Eq. (4) there remains

$$C_1 u + p = C_2,$$
$$\tfrac{1}{2}C_1 u^2 + (c_p/R)pu = C_3,$$

so that for each field variable a pair of values result. These are[3]

$$u_{+,-} = \frac{\gamma}{1+\gamma}\frac{C_2}{C_1} \pm \sqrt{(Rad)},$$

$$p_{+,-} = \frac{1}{1+\gamma}C_2 \mp \sqrt{(Rad)},$$

$$\rho_{+,-} = \frac{1}{2}\frac{\gamma+1}{\gamma-1}\frac{C_1^2}{C_3}\left(\frac{\gamma}{1+\gamma}\frac{C_2}{C_1} \mp \sqrt{(Rad)}\right),$$

$$Rad = \left(\frac{\gamma}{\gamma+1}\right)^2\left(\frac{C_2}{C_1}\right)^2 - 2\frac{\gamma-1}{\gamma+1}\frac{C_3}{C_1}.$$

All upper signs or all lower signs have to be taken simultaneously. Certainly a possible flow field is to let u, p, and ρ assume the values arising from the upper choice of signs for $x<0$ and the values arising from the lower choice of signs for $x>0$. This is exactly the situation corresponding to a shock discontinuity at $x=0$.

Since the Eqs. (4)–(6) are fulfilled by u_+, p_+, and ρ_+, and also by u_-, p_-, and ρ_- for the same values C_1, C_2, and C_3, they are the familiar shock conditions connecting the flow variables on both sides of a shock. In other words C_1, C_2, and C_3 are the constants characterizing the two flows and hence the discontinuity connecting them.

We see in this way that the equation system (4)–(6) contains in it a dependence of u, p, and ρ upon x which is intermediate in degree of complexity between the trivial case of constant values for u, p, and ρ and the case where u, p, and ρ turn out to be functions of x. This latter situation does not arise until viscosity and heat conduction are taken into account.[4]

III. THE SHOCK-STRENGTH PARAMETER

Inasmuch as C_1, C_2, and C_3 are the same on both sides of the shock, it is clear that a desirable strength parameter should be a function of a dimensionless combination of these constants. It can be seen readily that only one such combination exists. Since

$$C_1 = L^{-2}MT^{-1}, \quad C_2 = L^{-1}MT^{-2}, \quad C_3 = MT^{-3},$$

we construct $C_1^\alpha C_2^\beta C_3^\gamma$ and ask that it be a pure number. This leads to the linear homogeneous system

$$2\alpha+\beta=0,$$
$$\alpha+\beta+\gamma=0,$$
$$\alpha+2\beta+3\gamma=0,$$

whose only nontrivial solution is

$$\alpha:\beta:\gamma :: 1:-2:1.$$

Hence it is established that

$$C_1C_3/C_2^2$$

(or powers, or nondimensional functions thereof) is the proper parameter to characterize shocks. Denoting this quantity by N and using Eqs. (4)–(6) we note the following connection between it and the flow variables:

$$N = \frac{\tfrac{1}{2} + (\gamma/\gamma-1)(p/\rho u^2)}{[1+(p/\rho u^2)]^2}.$$

[3] Concerning γ see remarks in footnote 5.

[4] R. Becker, Z. Physik **8**, 321 (1922).

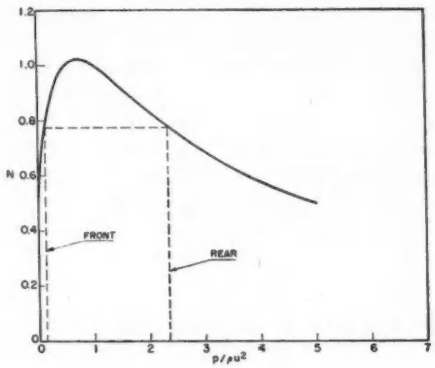

FIG. 1.

Here u, p, and ρ signify these quantities on either side of the shock; the same N value is attained for two values of $p/\rho u^2$. Figure 1 shows N as a function of $p/\rho u^2$ for a diatomic gas. The curve starts out with the value $\frac{1}{2}$, attains the maximum value of $\gamma^2/2(\gamma^2-1)$ for the abscissa γ^{-1} and becomes $\frac{1}{2}$ again for the abscissa value of $2/(\gamma-1)$ (which incidentally equals the number of degrees of freedom of the molecule).[5]

The curve of Fig. 1 indicates readily the flow parameters which obtain in front or behind a shock by intersecting the N curve by a horizontal line. Thus the limiting case of the sound wave belongs to the maximum of the curve while the limit of the infinitely strong shock corresponds to the lowest points of the curve. As the figure indicates, the front lies on the side left of the maximum with a higher u and lower p, and the rear on the right of the maximum. Points on the curve beyond $p/\rho u^2=2/(\gamma-1)$ belong to flows which do not allow a shock discontinuity.

IV. OTHER SHOCK PARAMETERS

We shall now discuss briefly other shock parameters mentioned in Courant and Friedrichs.[1]

The ratio of the pressure p_- behind the shock to the pressure p_+ in front, often denoted by y or ξ^{-1}, has the convenient property of varying from unity for sound waves to infinity for infinitely strong shocks. The relation connecting our N with y is

$$N=\frac{1}{2\mu}\frac{(y+\mu)(\mu y+1)}{(y+1)^2}.$$

The density ratio $\rho_-/\rho_+=s$ varies only between finite limits. The quantity N can be written in terms of s if one uses the Rankine-Hugoniot equation,

$$ys-\mu(y-s)-1=0,$$

and it turns out to be

$$N=\frac{(1+\mu)^2}{2\mu}\frac{s}{(1+s)^2}.$$

The Mach number M of the flow can readily be connected to N if one notes that

$$\rho u^2/p=\gamma M^2.$$

Hence

$$N=M^2\frac{\frac{1}{2}M^2+[1/(\gamma-1)]}{[M^2+(1/\gamma)]^2}.$$

This expression, when regarded as a quadratic equation for M, gives the two values of the Mach number on either side. The relations between the "experimental" quantities p_-/p_+, ρ_-/ρ_+ or M and the fundamental flow invariant are comparatively complicated.

The quantity N has as a disadvantageous feature the limited range

$$\tfrac{1}{2}\leqslant N\leqslant\tfrac{1}{2}(\gamma^2/\gamma^2-1).$$

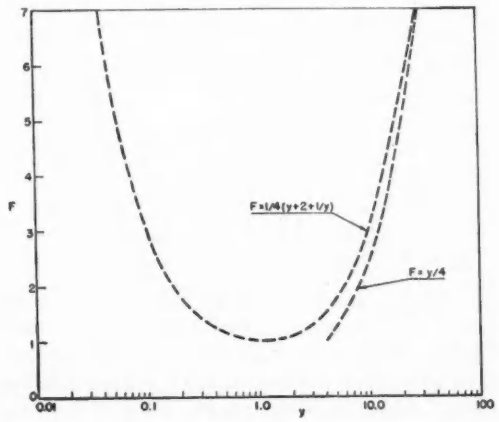

FIG. 2.

[5] One may note that there are at least four notations for the parameter characterizing the degrees of freedom of an ideal gas. Here the notation $c_p/c_v=\gamma$ is used, but also $\mu=(\gamma+1)/(\gamma-1)=f+1$ is employed. Here, f is equal to the number of degrees of freedom of the gas. Courant and Friedrichs use $\mu^2=(\gamma-1)/(\gamma+1)$.

It is therefore desirable to construct a simple function of N which, like p_-/p_+, will be unity for sound waves and will approach infinity for infinitely strong shocks. As such a function the following is the simplest possible:

$$F=\frac{\frac{1}{2}\gamma^2/(\gamma^2-1)-\frac{1}{2}}{N-\frac{1}{2}}.$$

It is this parameter which we wish to suggest as a most convenient invariant parameter. Its connection with the pressure ratio y,

$$F=\tfrac{1}{4}[y+2+(1/y)],$$

is very simple indeed. It has the advantage of being independent of γ, it does not distinguish between front and rear of the shock, and for strong shocks it becomes approximately proportional to y. It therefore shares with y the property of entering in a simple fashion into many relations concerning the strong shocks. Figure 2 shows F as a function of y, and Table I shows the numerical values of several of the shock parameters discussed in this section for a number of shocks in a diatomic gas.

TABLE I. Shock parameters.

y	M_+	M_-	ρ_-/ρ_+	N	F	$1/y$
1	1	1	1	1.042	1	1
5	2.104	0.561	2.82	0.789	1.8	0.2
10	2.952	0.478	3.81	0.672	3.025	0.1
20	4.158	0.431	4.65	0.5945	5.5125	0.05
50	6.557	0.400	5.38	0.540	13.005	0.01
∞	∞	0.3779	6.00	0.500	∞	0

The Tape Recordings of Important Speeches

BY

E. U. CONDON, K. K. DARROW, E. FERMI, AND J. C. SLATER

Free tape recordings are available on loan to members of the Association who wish to use them for presentation to classes, seminars, or science clubs. The subjects are four of the six lectures presented as a Symposium on Physics Today at the Twentieth Anniversary meeting of the American Institute of Physics, Chicago, Illinois, on October 25, 1951, as follows:

The atom. E. U. CONDON.

Physics as science and art. K. K. DARROW.

The nucleus. E. FERMI.

The solid state. J. C. SLATER.

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Training of High School Physics Teachers and Cooperation between College Physics Departments and High Schools

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(Received April 16, 1953)

A report is presented of a study made primarily in the Southern States to determine what the certification requirements for teachers of high school physics are and what some college physics teachers think they should be; to determine what special undergraduate and graduate programs of study for those preparing to teach or already teaching high school physics are provided by colleges; and also to discover to what extent college physics departments are cooperating with high school physics departments, by loaning equipment, giving assistance to science clubs, presenting demonstration lectures at high school assembly programs, and discussing opportunities for a career in physics with high school seniors.

MANY articles have been written during the past few years expressing concern about the manpower shortage in the fields of science, engineering, and mathematics.[1] Most of the emphasis in these articles has been placed upon the needs of government and industry. Nothing much has been written about the needs of the high school teacher of science with respect to his preparation and the modernization of his subject-matter knowledge.

The author has become rather concerned over where our physics teachers, particularly for the secondary schools, are going to come from in the future. He is also interested in what is being done to improve physics teaching on the secondary level. In an effort to obtain more information about these problems, and also the opinions of other college physics teachers, a questionnaire was sent out to the heads of physics departments in 31 universities and large colleges, 35 small liberal arts colleges, and 32 teachers colleges, principally in the Southern States. This paper is an attempt to report and to evaluate the findings from the 53 questionnaires which were returned.

The first questions dealt with certification requirements for teachers of physics in the secondary schools. It was found that there was a variation of from 6 to 24 semester hours of physics required, with an average of about 10 hours. For the most part, it was felt that these requirements were reasonably enforced, although in a few states the replies indicated that they were definitely not enforced. Several states also granted temporary certificates without obliging the teachers to meet the minimum requirements. Suggestions from the heads of college physics deqartments as to minimum requirements for certification to teach physics in high schools varied from 10 to 24 semester hours with 16 hours being suggested by a majority.

One question concerned the number of physics, or physical science, majors who had become physics or science teachers. The answers were rather difficult to interpret in many cases, but apparently very few physics or physical science majors enter the teaching field from the universities and liberal arts colleges, and even among the teachers colleges the percentage becoming teachers is often quite small. Most heads of departments indicated that in some way they had tried to encourage at least some students to enter the teaching profession, but several reported that they even discouraged students from entering the profession, principally because of the low salaries.

Questions were asked regarding the nature of undergraduate physics courses required of and offered for prospective high school physics teachers. Most college departments reported that they offer only the usual basic and intermediate courses, but some of the teachers' college departments offer a special course in "Methods." Several respondents, however, indicated that they thought certain types of special courses might well be offered for prospective teachers. Some suggestions were as follows: intermediate

[1] R. B. Lindsay, Sci. Monthly **74**, 139 (1952); M. H. Trytten, J. Eng. Educ. **42**, 77 (1951); J. H. Lux and L. S. Moody, Chem. and Eng. News **29**, 5330 (1951); A. V. Astin, Education **73**, 405 (1953).

courses which would be less mathematical than usual; general courses with special emphasis on applications of physics and topics of special interest to high school pupils; courses in the history and philosophy of physics; methods courses in demonstration lectures, laboratory techniques, apparatus making and repairing, and visual aids; and finally, content courses (but not too technical) in photography, electronics, radio and atomic and nuclear physics.

It was found that very few of the colleges with graduate division offered any special graduate courses or degrees designed especially for high school physics teachers. The majority of department heads felt that this situation needs attention, and some college departments are planning to do something about it. It was suggested by twelve respondents that courses for graduate work should be similar in nature to advanced undergraduate courses, with special stress placed on the practical side.

The author feels very strongly that this is a matter which demands immediate attention. It appears from the results of the questionnaire that most of the present high school physics teachers have had only one course or at most only two courses in physics. With that background, the teachers are not qualified to take graduate work in physics in the usual sense, but it is felt that special courses, and special degrees, if necessary, should be provided and teachers should be encouraged to take additional work in physics, expecially in the new fields of electronics and atomic physics with which few of them are familiar. Many states provide salary increases for those teachers who obtain Master's degrees, but at present there are only a few colleges which offer a Master's degree which even includes physics as a possible minor subject.

The answers to another section of the questionnaire seem to indicate that experimentation in organizing refresher courses and workshops in physics and the other sciences would be beneficial, perhaps the best time being during the summer months. Such courses or workshops might also be offered for graduate credit. Very few colleges at present even offer such opportunities for high school teachers, but again some have indicated that they plan to do this.

The author was pleasantly surprised to find that the college physics departments are contributing quite a bit to the high school physics teachers and students by loaning equipment, giving demonstration lectures in high school assembly programs or to science clubs, speaking at "Career Days," holding "Open Houses" or "Science Fairs," and acting as advisors to Junior Academies of Science groups.

Most of the college departments have done little to seek cooperation with high school physics teachers through group conferences; and those doing so indicated that the attendance was usually poor, with little interest and very limited accomplishments. However, a few experienced reasonably good results from such conferences, and some indicated that they had worked successfully with individual teachers.

The respondents were practically unanimous in their opinion that more cooperation between college and high school physics departments would be beneficial especially on a person-to-person basis. The results of these contacts would be numerous. High school physics instruction would be improved by suggestions that the college teachers might make. High school teachers might be stimulated to take refresher and graduate courses in physics or attend workshops. Contact of the college teachers with the better high school pupils would provide opportunities to discuss a career in physics with them. The general public would become better informed about the nature of physics and some of its practical uses. College teachers would obtain insight into the high school teachers' problems which should resultingly improve the college training of future teachers. And finally, a genuine comradeship of those with mutual interests and problems would develop. Some emphasize that when the college teaching load is computed, time spent helping high school teachers and pupils should be counted.

In conclusion, the author would like to make a few suggestions to college physics teachers which have come to mind while perusing the questionnaires:

1. The head of each college physics department should appoint one member of his department to supervise cooperation with high school physics teachers and departments. Such a person

should probably belong to the National Science Teachers Association and any state organization of science teachers; in turn, he should urge high school physics teachers to join the American Association of Physics Teachers. He should visit high school physics teachers in his geographic area, volunteering any help that he, or his department, may be able to give. He should encourage teachers in their professional growth; he should attempt to discover pupils with the proper scientific abilities and encourage them to become physicists. He should offer his services helping to establish Junior Academies of Science, serving as adviser to them, and helping with the Science Talent Search.

2. Each college physics department should try to hold an annual "Open House" or cooperate with other science departments in sponsoring a "Science Day" or "Science Fair."

3. Each department should try to keep in touch with former students who may be high school physics teachers and should try to help them in every possible way.

4. Each department should volunteer speakers or counselors for high school "Career Days."

5. Each department should try to contact the local school superintendent in its area and urge that only the best qualified science teachers be appointed to the schools.

6. College physics departments, either individually or collectively, should urge the State Boards of Education to raise certification requirements for high school science teachers who teach physics.

7. College physics departments should urge State Boards of Education to require that teachers of high school physics take physics courses to fulfill requirements for renewal of teaching certificates.

8. College physics departments should offer refresher courses and workshops for high school science teachers.

9. At least one college in each state should offer a special curriculum of graduate work in physics or a combined curriculum in the physical sciences for high school physical science teachers, the courses of which will be especially designed for high school teachers and which will lead to a master's degree.

10. College physics departments should ask for representation at discussions pertaining to the physical science courses offered in the high school curriculum.

11. College physics departments should seek the cooperation of industry, in whatever way possible, to further improve physical science programs in the secondary schools.

The author hopes that some of the ideas expressed will stimulate college physics teachers to think more about and to help find solutions for some of the problems indicated in this discussion. Many may feel that the problems of the high school physics teacher are beneath the dignity, or outside the province of, the college physics teacher, but such persons may find unexpected satisfaction if they are willing to take the time and put forth the effort to do something constructive in this area. Physics has grown very rapidly during the past half century and we are proud of its accomplishments, but many of the citizens of the United States still do not know what physics is about or realize its importance in their future. Apparently, even in the educational realm, many teachers and administrators are still unaware of its vast significance; so let's do something about it!

Making the Laboratory a More Effective Scientific Experience

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(Received April 28, 1953)

Suggestions are made for restoring to elementary laboratory work the sense of adventure and intellectual excitement which has to a considerable extent been displaced by the notion that "efficient" and assembly-line speed of instruction is desirable.

TO improve the effectiveness of the elementary laboratory, we introduced some features that are belived to minimize or eliminate certain shortcomings of the laboratory as it is traditionally operated. The aims were to afford an opportunity for experiences and feelings about experimental work that cannot be had through lecture-demonstrations, and, at the same time, to avoid the artificiality of stereotyped and disconnected laboratory "exercises."

The late Professor Richtmyer[1] described a course of intermediate or advanced grade having such aims, in which the student was assigned an experimental problem that engaged his time for the entire semester. The problem might be, for instance, to determine many of the physical properties of a particular substance. G. B. Welch,[2] of Bowdoin College, instituted a similar course; but it, too, was designed as a second course. F. D. Cruikshank,[3] of the University of Tasmania, reported on a course of this character that occupied the first two years.

Thus, there is nothing novel about the essential features of our plan. It is simply an attempt to restore to laboratory instruction some features that have been neglected or lost since the advent of mass education, namely, the opportunity for the student to work from the very start in a real laboratory atmosphere and under apprentice-like conditions. The resulting effect on student interest and attitude can be very appreciable; and this is important, since it is in the first-year course that the interest of so many of the students is either gained or lost. Moreover, it is more desirable today than ever that undergraduates have the opportunity to engage in individual semi-creative work in the laboratory, since many of them who go on to graduate study will gain experimental experience mainly as members of large research teams.

The procedure we followed attempts to simulate actual laboratory working conditions by assigning to the students several problems of fairly long range, each requiring from two or three weeks to as much as six or seven for completion. The student is told what is to be determined and the various kinds of pertinent apparatus that are locally available. But the particular method of attack and details of experimentation are left to his discretion. He begins with some library search which, to keep it from becoming too involved and time-consuming, should usually be confined to selected reference books kept on a special reserve shelf in the physics library or the laboratory. When possible, the laboratory should be kept open at times other than scheduled periods, for the convenience of those who wish to do additional work.

If there is to be a feeling of accomplishment and satisfaction in the use of equipment, the problems should be of such character that there will be frequent repetition in the use of many of the instruments and apparatus. One is not likely to gain much of a feeling of accomplishment and growth if, each week, he encounters apparatus that is almost entirely new and unfamiliar. He is always in the position of being, so to speak, a beginner. Moreover, the more he feels at home with a piece of equipment, the more confidence he is likely to have in the significance of his results and in his ability to evaluate them. The ideal plan, in this respect, would be to issue to the student a small set of equipment for his exclusive use throughout the course—meter stick, ruler,

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[1] Committee on the Teaching of Physics for Premedical Students, Am. J. Phys. (Am. Phys. Teacher) **2**, 104 (1934).
[2] G. B. Welch, Am. J. Phys. (Am. Phys. Teacher) **3**, 69 (1935).
[3] F. D. Cruikshank, Am. J. Phys. **17**, 15 (1949).

calipers, set of weights, spirit level, small set of hand tools (especially pliers and screw driver), Bunsen burner, thermometer, and so on. This is the plan usually followed in elementary chemistry. Yet it should be still more effective in physics, where there are relatively so many instruments with peculiarities that have to be learned and many that need frequent recalibration.

The teaching of the use of such measuring devices as vernier and micrometer calipers, and the calibration of such equipment as weights and thermometers, naturally should be left as an incidental part of the over-all problem. One learns to use or to calibrate them as the need arises, not as exercises in themselves.

Weekly laboratory reports of the conventional sort are not required. Instead, the student submits a brief progress report, which includes a statement about his tentative plans for the week to follow. Then, upon completion of a particular problem, he prepares a final report that is patterned after the kind and style of paper that might be written for a periodical. It covers theories, procedures, results and evaluation of results; contains references to the literature and necessary diagrams or even photographs; and might well include suggestions for improved techniques and equipment, and for further work that could be done on the project. One of several advantages of this scheme is that the student is not encouraged in the notion that a finished, final report should be forthcoming after only a couple of hours of experimentation involving possibly unfamiliar apparatus and techniques.

Most college laboratories have a sufficient amount and variety of equipment to permit the assignment of a number of different problems. One problem that we have used is to make a comparison of the Wheatstone bridge and ammeter-voltmeter methods of determining the resistivity of copper; it involves, among other things, calibrations of the bridge, ammeter, and voltmeter, and the construction of a slide-wire potentiometer. Another problem is to determine, for a particular metal, the density, elastic constants, linear expansivity, specific heat, and speed of sound; and this list, of course, could be extended to include electrical and optical properties such as resistivity, electrochemical equivalent, and optical spectrum. Various physical properties of air or some other gas can be similarly investigated. Still another example is to compare the actual and ideal mechanical advantages, and efficiencies of a number of machines of different kinds.

In instituting such a laboratory course, the instructor faces the task of adapting, for use with a large group of beginners, a supervisory method similar to that employed with students engaged in individual research. In some institutions, this method of supervision, involving the use of individual long-range problems, literature research and reporting, has not been extended even to small groups in intermediate and advanced laboratories, the tendency often being to handle them by the same mass-education methods as are employed in the elementary laboratory. It should therefore be recognized that the extension of the plan advocated here to large groups of beginners is something requiring considerable time and patience to learn; otherwise, there will be an understandable temptation to let the course gradually revert to the traditional type, with consequent loss of the opportunity to see what might have been gained by continuing the new plan. If a director of that new methodological phenomenon, the large research team, were to give up trying to learn his new and difficult task, the loss in productivity of the team would soon become apparent.

It may be objected that the new type of course makes unreasonable demands on the energy and time of the instructor. This is likely to be true at the beginning. But it is our experience that the instructor who continues in his efforts will find the new course gradually becoming little more difficult to administer than the traditional one.

Demonstration of Parabolic Velocity Distribution in Laminar Flow

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(Received February 19, 1953)

This paper describes a method for rendering visible the parabolic velocity distribution in laminar flow through a conduit. An opaque cloud having the shape of a thin line disposed along a pipe diameter is created by electrochemical means. This line is distorted into a parabola by the motion of the fluid.

IN view of the scarcity of quantitative lecture demonstrations and laboratory exercises in fluid dynamics, it is hoped that the experiment described below will be a welcome addition to the available teaching devices.

The objective of this experiment is to demonstrate the parabolic velocity distribution in laminar flow of a viscous fluid through a conduit qualitatively by inspection and quantitatively by a graphical analysis of the observation.

The velocity distribution throughout the cross section of a circular[1] pipe is demonstrated by creating a linear cloud of foreign, easily observable material along a diameter of the cross section while the fluid is at rest. The laminar flow distorts the cloud by carrying its central portion farther downstream than the parts proximal to the pipe wall. Thus, the originally linear cloud W is deformed into a parabola P as shown in Fig. 1A.

Figure 2 illustrates the main part of the apparatus. It consists of a block of Lucite of rectangular cross section traversed by a circular channel through which the viscous fluid is allowed to flow. Plastic end pieces E_1 and E_2 are screwed into the tapped ends of the channel. They serve to connect the Lucite pipe to the reservoir and to the vessel collecting the outflowing liquid. Copper wires W and C (gauge Nos. 26 and 16, respectively) are attached to screws S_1 and S_2. The latter can be easily removed, thus making it possible to clean the wires W and C.

Two methods have been used to produce the opaque cloud. In the first one a viscous mixture of glycerine with a solution of acidulated potassium iodide containing analytic starch reagent is used. (The liquid is composed of: 84.2 percent glycerine, 15 percent water, 0.5 percent KI, and 0.3 percent soluble starch by weight. The starch is dissolved by boiling it in the KI solution before mixing with glycerine. Subsequently one drop of 35 percent HCl is added per 10 cc of solution.)

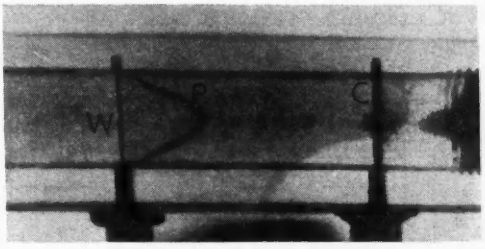

FIG. 1A. Distortion of the linear starch-iodine cloud, originally surrounding the wire W, into a parabolic cloud P. Wire W serves as anode and C as cathode.

FIG. 1B. The original cloud surrounding the wire is allowed to rise above the wire by thermal convection before the fluid is set in motion. A parabolic area is thus seen in projection after a brief period of flow.

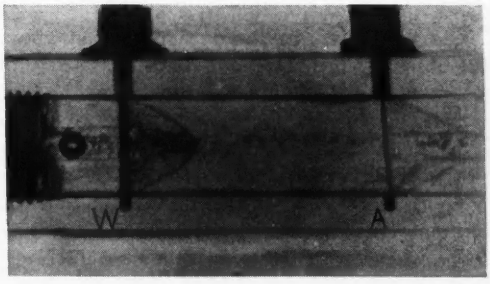

FIG. 1C. Visualization of the parabolic velocity profile by means of the gas bubbles developed at the wire W which serves as cathode, while the wire A is used as anode.

[1] A conduit of rectangular cross section can be used just as well.

The electrodes W and C are connected to a dc power source. The iodine liberated at the anode W reacts with the starch forming a deep blue linear cloud. After passing a current of about 20 ma for approximately 4 seconds one may allow the fluid to flow, stopping it when the blue cloud has suffered the desired deformation (Fig. 1A).

If the current is passed for a longer time interval, the blue cloud rises upward from the horizontal wire because of thermal convection and a deep blue semicircular "curtain" is formed above the wire. When the fluid is set in motion, this semicircular area is distorted into a semiparaboloid which, in projection, appears as a solid parabolic area (Fig. 1B).

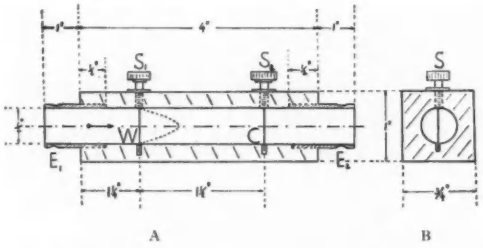

FIG. 2. (A) Longitudinal cross section of the Lucite pipe. E_1 and E_2: end pieces for attachment of rubber tubing; S_1 and S_2: flanged screws to which are soldered the wires W and C (the rubber gasket between the flange and the Lucite wall is not shown); W and C: electrodes. (B) Transverse cross section of the Lucite pipe.

The second method is much simpler, although of somewhat lesser contrast. It uses the rising hydrogen bubbles liberated at the wire W of Fig. 1C, which serves as the cathode, to form the semicircular "curtain." The parabola thus obtained after a brief period of flow is shown in Fig. 1C.

The parabolas produced as described above can be easily projected and photographed. For best results, it is recommended to orient the pipe so that the wire W is horizontal using a vertical projector. This minimizes the danger of distorting the parabola by convection caused by density difference between the "cloud material" and the solution.

The velocity distribution predicted theoretically is represented by the equation:

$$V_0 - V = V_0 R^2 / R_0^2,$$

where V_0 is the maximum velocity (along the pipe axis), R_0 is the pipe radius, R the distance of a given point from the pipe axis, and V the velocity at that point. By plotting R^2 as a function of $(V_0 - V)$, which is measured by the distance of the given point on the parabola from the tangent at the apex, the above parabolic relation can indeed be verified by the student as shown in Fig. 3.[2]

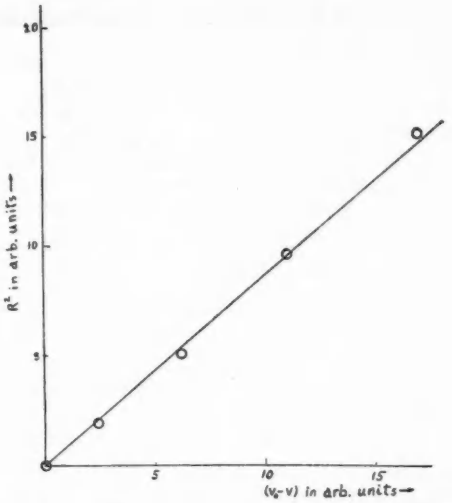

FIG. 3. Verification of the equation $(V_0 - V) = V_0 R^2 / R_0^2$, relating the local velocity V to the axial distance R. (V_0 is the velocity along the pipe axis and R_0 the pipe radius.)

The flow was not a steady one in these experiments. The distance between the wire W and the rim of the parabola is a measure of the average velocity of flow. These experiments demonstrate that the average velocity in nonsteady laminar flow is distributed in parabolic fashion just as the instantaneous velocity in steady laminar flow.

The author wishes to thank Mr. D. G. Oldfield for technical assistance.

[2] The indices of refraction of the KI-Glycerine solution ($n = 1.45$) and of the Lucite ($n = 1.49$) are quite close. The parabolicity of the photographed curve is not noticeably affected by refraction, at least in the central region extending over about 70 percent of the pipe diameter.

A Geometric Approach to Nonrelativistic Spin Theory

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The requirements that must be satisfied by a nonrelativistic theory of electron spin are examined, and it is shown, with the aid of elementary spinor theory, that these requirements can be met by a geometric scheme whose main operation is the reflection of spinors in an axis. This yields a kind of picture of electron spin. The analytical formulation of the geometric scheme is developed, and is identical with the Pauli spin theory. The part played by nonstationary spin states in the geometric scheme is discussed.

1. INTRODUCTION

THE object of this article is to show how the nonrelativistic (or Pauli) theory of electron spin[1] can be set up in terms of purely geometric operations. This approach to spin theory is more specific and more elementary than the usual formalistic treatment, and it permits the use of diagrams to aid in the visualization of the operations. The treatment to be given here involves no new physical hypotheses; all of it is contained implicitly in the usual formulation of nonrelativistic spin theory, to which it is mathematically equivalent.

Since most present day theoretical physicists are accustomed to formalistic rather than geometric methods, the present approach may seem to some to be insufficiently general. The formalistic approach to spin theory is based on certain postulated algebraic rules involving the fundamental quantities (i.e., the Pauli spin operators), and these rules are entirely independent of the choice of coordinate system or other conventions used. The geometric approach, on the other hand, might seem at first sight to be committed to one specific set of conventions; thus in Fig. 1 the coordinate system and the positive senses and base lines of the angles have all been chosen arbitrarily, and in Eqs. (1) we have arbitrarily selected one particular way of defining the components of a spinor out of many possible ways. [Another possible way is given by Eqs. (8).] However, the further development shows that there is really no *essential* lack of generality in the geometric scheme, for the spin energy operator (which is the main *physical* quantity in the scheme) is defined (Sec. 4) in terms of a geometric operation that depends in no way whatever on the choice of conventions.

The mathematical apparatus required for the present treatment is elementary spinor theory.

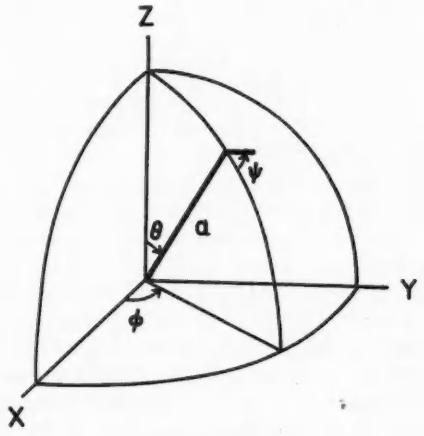

FIG. 1. Spinor.

This is a purely mathematical scheme and not, as the similarity between the words "spin" and "spinor" might suggest, a branch of physics. Spinor theory is usually developed from a consideration of the representations of the group of rotations in three-dimensional space, but there is a much more elementary approach to spinor theory,[2] and this will be used in the present article.

[1] Discussions of the Pauli spin theory may be found in most books on quantum mechanics. See also *Handbuch der Physik* (Springer, Berlin, 1933), Vol. 24/1, articles by W. Pauli and by H. Bethe; E. L. Hill and R. Landshoff, Revs. Modern Phys. **10**, 87 (1938); R. B. Lindsay and H. Margenau, *Foundations of Physics* (John Wiley and Sons, Inc., New York, 1936), Sec. 9.13. The development to be given here is most nearly like that in H. A. Kramers, *Quantentheorie des Elektrons und der Strahlung* (Akademische Verlagsgesellschaft, Leipzig, 1938), pp. 239–246.

[2] W. T. Payne, Am. J. Phys. **20**, 253 (1952).

It may be remarked here that the applicability of spinor theory is by no means restricted to spin or to quantum phenomena, for it can be applied to four-terminal ac networks[3] and possibly to other classical problems.

2. BASIC HYPOTHESES

The hypotheses on which we shall consider nonrelativistic spin theory to be based are as follows:

1. An electron has an angular momentum of magnitude $\frac{1}{2}\hbar$.

2. An electron has a magnetic moment of one Bohr magneton (i.e., $e\hbar/2mc$), whose direction is opposite to that of the angular momentum just mentioned.

3. When an electron is in an external magnetostatic field, its angular momentum vector will always be found to be either parallel[4] or antiparallel to the field, never in any other direction.

The first two of these are just the hypotheses laid down by Goudsmit and Uhlenbeck in their original work on electron spin. The third, which is suggested by the Stern-Gerlach experiment, is not usually listed explicitly as a separate hypothesis, but is considered to follow from the first and from the quantum principle that successive realizable values of a given component of the angular momentum of a dynamical system must differ by $\hbar$. (Here the dynamical system is the electron in the external magnetic field; the given component of the angular momentum is the component in the direction of the magnetic field; and the realizable values of this component are $\frac{1}{2}\hbar$ and $-\frac{1}{2}\hbar$). Thus our three hypotheses are not all logically independent of one another, though it is convenient, for present purposes, to write them down as three separate items.[5]

Bohr has given reasons why spin phenomena cannot be experimentally observed on free electrons because of the way in which the uncertainty principle works.[6] This limitation, however, probably does not damage the arguments to be used here, which, after all, are mathematically equivalent to those used in every treatment of Pauli spin theory.

The problem of building up a complete nonrelativistic spin theory on the basic hypotheses can be attacked in three main ways, as follows.

First, one can assume a model for the electron, e.g., a charged, rotating, rigid sphere, and treat it according to the laws of classical dynamics.[7] This leads to a conflict with the third hypothesis because, according to classical mechanics, a charged rotating body in an external magnetic field could have its angular momentum vector directed arbitrarily and the only effect of the field would be to make this vector precess about the direction of the field.

Second, one can again assume a model and treat it according to the rules of quantum mechanics, applied straightforwardly (i.e., by substituting differential operators for the generalized momenta in the classical energy equation).[8] This method leads to the incorrect conclusion that the electron in a magnetic field must have an infinite number of stationary states instead of only two, as required by the third hypothesis.

Third, one can try to devise a mathematical scheme that shall satisfy the basic hypotheses completely and the rules of quantum mechanics at least partly. The Pauli theory is such a scheme. Although quantum mechanical, it avoids the difficulties mentioned in the preceding paragraph. Mathematically, the Pauli theory is an adaptation of Hamilton's quaternion theory, and, like the latter, is usually formulated in terms of abstract operators and the multiplication rules that they satisfy. We shall set it up, however, in terms of geometrical operations on spinors. It may be asked why spinors (rather than, for example, vectors) should have to be used. The reason is that in spinor theory there are just two basis elements, which are antiparallel to each other, and these can be associated with the two possible situations stipulated by the third hypothesis. Ordinary vector theory would not fit the problem because it is built up on *three* basis elements (i.e., the unit vectors **i**, **j**, **k**) that are *perpendicular* to one another.

[3] W. T. Payne, J. Math. Phys. **32**, 19 (1953).

[4] Throughout this paper, the word "parallel" means having the same *sense* as well as the same direction.

[5] Anyone interested in seeing what the minimum hypothetical basis for spin theory is should consult J. von Neumann and E. Wigner, Z. Physik **47**, 203 (1927).

[6] *Handbuch der Physik* (Springer, Berlin, 1933), Vol. 24/1, p. 242.

[7] H. A. Kramers, reference 1, pp. 227–239. Lindsay and Margenau, reference 1.

[8] C. G. Darwin, Proc. Roy. Soc. (London) **115A**, 1 (1927). For a more recent attempt, see N. Rosen, Phys. Rev. **82**, 621 (1951).

3. FUNDAMENTALS OF SPINOR THEORY

A spinor has magnitude (a) and three Eulerian angles (θ, φ, and ψ), as shown in Fig. 1. We shall call the radial segment of the spinor the handle and the tangential segment the blade. The magnitude a need not have the dimension of length but can, like the magnitude of a vector, have any desired dimensionality; in the present application it has to be dimensionless. Analytical spinor theory is based on two complex components a_1 and a_2, defined as follows,

$$\left.\begin{aligned} a_1 &= (2a)^{\frac{1}{2}} \cos\tfrac{1}{2}\theta e^{i(\psi+\varphi)/2} \\ a_2 &= (2a)^{\frac{1}{2}} \sin\tfrac{1}{2}\theta e^{i(\psi-\varphi)/2} \end{aligned}\right\}. \qquad (1)$$

The rotation of a spinor through 360° about any arbitrary axis will cause each of its components to be multiplied by -1. To see this,[9] note that if the z axis is inside the cone traced out during the rotation by the spinor handle, then θ and ψ will come back to their initial values, and φ will be changed by 360°, so that, according to Eqs. (1), a_1 and a_2 will be multiplied by $e^{\pm\pi i}$, or -1. If, on the other hand, the z axis is outside the cone, then θ and φ will come back to their initial values, and ψ will change by 360°, and a_1 and a_2 will again be multiplied by -1.

We must now define an operation that we shall call reflection of a spinor in an axis. Here an axis is understood to mean a half line going out from the origin. To reflect a spinor in a given axis, rotate the spinor until its handle lies along the axis and then rotate it an equal angular distance farther, in such a way that the spinor handle traces out a section of a plane (rather than a section of a conical surface). This is illustrated in Fig. 2, in which $\mathbf{a}$ and $\mathbf{a}'$ are the initial and final positions of the spinor, the dotted radius is the axis of the reflection, and the dotted great circle arc is the path along which the end of the handle moves. Note that the angle between the spinor blade and the dotted arc does not change. Note also that the reflection of all spinors in a given axis is not equivalent to the rigid rotation of all spinors about any one axis, since the reflection causes each spinor to be rotated about a different axis.

Since we are later going to use some of the machinery of quantum mechanics, we shall have to concern ourselves with the eigenspinors and eigenvalues of certain operators. Let us see what the eigenspinors and eigenvalues of a reflection in an axis will be. An eigenspinor of a given operator is defined as a spinor that, when acted on by the operator, will merely have both of its components multiplied by one and the same real or complex number, instead of being transformed in some more complicated way. In geometrical terms [as follows at once from Eqs. (1)], an eigenspinor is one that does not get its θ or its φ changed when the operator works on it, though its magnitude and its ψ may be changed. The eigenvalue of an operator

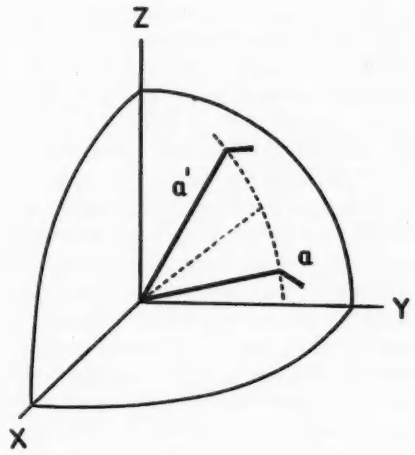

FIG. 2. Reflection of a spinor in an axis.

associated with a given eigenspinor is defined as the common factor by which the components of the eigenspinor are multiplied when the operator acts on it. From these definitions we can draw the following conclusions in regard to a reflection of spinors in an axis. First, any spinor whose handle lies along the axis of reflection is an eigenspinor of the reflection with the associated eigenvalue 1, since such a spinor is not changed in any way by the reflection. Second, any spinor antiparallel to the axis of reflection is an eigenspinor of the reflection with the associated eigenvalue -1, since the operation of reflection, as defined above, will cause such a spinor to be rotated, altogether, through 360°.

[9] A more formal proof is given in reference 2, Sec. 5.

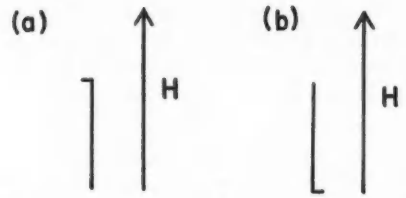

FIG. 3. Eigenspinors for an electron in a magnetic field **H**. (a) Negative energy state. (b) Positive energy state.

Third, a spinor that is neither parallel nor antiparallel to the axis of reflection is not an eigenspinor of the reflection, since the reflection will change its θ or its φ or both.

4. GEOMETRIC DEVELOPMENT OF SPIN THEORY

We can now state the mathematical problem involved in the Pauli spin theory and solve it geometrically. Quantum mechanics requires that the problem be set up in terms of operators, stationary states, eigenfunctions, and eigenvalues; and we shall expect the mathematical problem to consist in the finding of a suitable energy operator for the Schrödinger equation describing the electron's behavior in an external magnetic field. The third hypothesis suggests that the electron in a magnetic field should be considered to have only two stationary states, the parallel state and the antiparallel state. The values of the electron's energy in these two states ought to be the negatives of each other for classical reasons.[10] Accordingly, the mathematical problem can be stated as follows: for an electron in an external magnetic field **H**, we have to find an energy operator having two eigenfunctions that express, respectively, parallelism and antiparallelism of the electron's magnetic axis to **H**, and having two eigenvalues that are real and are the negatives of each other.

The solution of this problem is immediately evident, provided we agree to replace the word "eigenfunctions" by "eigenspinors." For the desired energy operator we can take the product of a suitable physical constant by a reflection of spinors in the axis of the magnetic field. The two independent eigenspinors[11] of this operator are, according to Sec. 3, parallel and antiparallel to the magnetic field, and the two eigenvalues are the negatives of each other. The physical constant should be taken to be $e\hbar H/2mc$ (where H is the magnitude of the magnetic field) for then the two eigenvalues for the energy will be $\pm e\hbar H/2mc$, as they should be according to the second hypothesis and the classical formula for the energy of a magnetic dipole in a magnetic field.[10]

A small revision is necessary at this point. Instead of taking the energy operator to be (except for a positive constant) a reflection in the axis of the field **H**, we could just as well take it to be a reflection in the axis antiparallel to **H**. The only effect of this change would be to reverse the association between the two energy values and the two eigenspinors, so that the negative energy value would now go with the spinor parallel to **H**. Which of the two procedures to use is purely a matter of convention, and, as it turns out, agreement with the usual conventions is attained by taking the energy operator to be $e\hbar H/2mc$ times reflection in the axis *antiparallel* to **H**. This we shall do.

In one of the stationary states, then [Fig. 3(a)], the eigenspinor is parallel to **H** and the energy is negative, and therefore in this state the electron's magnetic moment must be parallel to **H**,[10] and so its angular momentum must be antiparallel to **H** by the second hypothesis. In the other stationary state the opposites of all these statements are true. In either state the electron's magnetic moment has the same direction and sense as the eigenspinor, and the electron's angular momentum has the opposite sense.

Most discussions of spin theory do not use the word "spinors" at all, but instead introduce two "spin functions," sometimes called α and β, to describe the spin. The external magnetic field **H** is usually assumed to be in the positive z direction; and the "spin functions" are defined as follows: one of them is unity when the electron's magnetic moment is parallel to **H** and zero when

[10] Classically the energy of a magnetic dipole of magnetic moment $\mathbf{p}$ in an external field **H** is $-\mathbf{p}\cdot\mathbf{H}$, if only the rotational degrees of freedom of the dipole are considered. See, for example, L. Page and N. I. Adams, *Principles of Electricity* (D. Van Nostrand Company, Inc., New York, 1949), second edition, p. 126.

[11] Note that there are only two independent eigenspinors of the reflection. Two spinors having the same θ and φ but different ψ are not independent, since, according to Eqs. (1), their components are proportional.

it is antiparallel to **H**, and the other has the values zero and unity in these situations. The two "spin functions" are identical with our two eigenspinors, the two values that each can assume being equal to the two components of the corresponding eigenspinor.

The use of a spinor to give a picture of the electron's spin properties is just as correct as is the familiar use of a de Broglie wave or a wave packet to give a picture of the wave properties of a particle. We should, however, note the following facts.

First, the magnitude of the spinor is not determined by the above described scheme, so it is quite arbitrary. Therefore there is no way of getting from the scheme any quantity that might be regarded as the radius of the electron.

Second, the only observable is the sense of the spinor. One might think that the angular position of the spinor blade (or, in other words, the value of the third Eulerian angle ψ) ought to be observable also. It is not observable, however, because ψ enters into the components of the spinor [Eqs. (1)] only as a common phase, and according to quantum mechanics the phase of a stationary state is not observable.

Third, each of the eigenspinors must rotate at uniform speed about its own axis because, according to quantum mechanics, the time dependence of a stationary state of a system is described by an exponential factor $e^{i\omega t}$, so that the spinor's phase angle $\frac{1}{2}\psi$ must be a linear function of the time. Here ω is $(1/\hbar)$ times the energy of the system in the given stationary state. The angular speed of the rotation is easily computed for the case where the electron has no energy except that due to its magnetic moment in the external magnetic field. In that case, ω is $eH/2mc$ for one state and $-eH/2mc$ for the other state, so that

$$d\psi/dt = 2\omega = \pm eH/mc. \qquad (2)$$

The two stationary state spinors rotate as if they were rigidly connected together, as is seen from Eq. (2) and from the way in which the sense of ψ has been defined (Fig. 1). According to Eq. (2), there is no rotation if **H** vanishes, a fact that is at variance with the popular idea of the spinning electron. If the electron has other energy besides its spin energy (and this is always the case in practice), then the two values of ω will not be the negatives of each other, and the behavior of the stationary state spinors is more complicated.

Since the spin states are described by a geometric object specified by three Eulerian angles, it follows that any other similarly specified geometric object would serve just as well as a picture of the electron, for example a rotating rigid sphere of unspecified radius. It may seen surprising, in view of the shortcomings of the older spin theories discussed in Sec. 2, that the rotating sphere can, after all, be correctly used as a model for electron spin. The essential point is that in the older theories the rotating sphere was taken to be the *classical* model for the electron, whereas here the rotating sphere is the *quantum-mechanical* model, playing the role in spin theory that corresponds to that of the de Broglie wave in ordinary quantum mechanics.

5. ANALYTICAL DEVELOPMENT OF SPIN THEORY

To get the analytical formulation of the geometric scheme described in Sec. 4, we must find the equations for the reflection of spinors in a given axis. This is most easily done by noting that the reflection is equivalent to a 180° rotation of spinors about the given axis followed by a 180° rotation of each individual spinor about its own axis. This is shown in Fig. 4, in which the dotted radius is the axis of reflection or rotation,

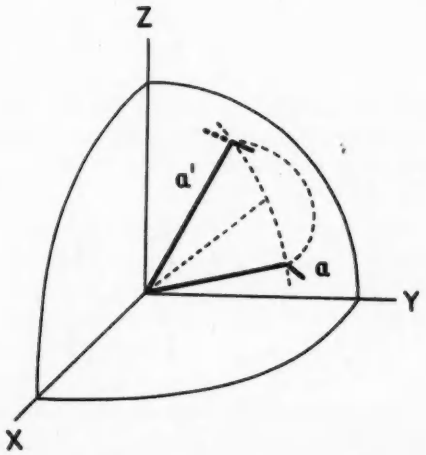

FIG. 4. Analysis of a reflection into rotations.

the two dotted arcs are the paths of the end of the spinor handle during reflection and 180° rotation, and $\mathbf{a}'$ marks the two final positions of the spinor, the one with the solid blade being obtained by reflection and the one with the dotted blade by 180° rotation. The two 180° rotations into which we have analyzed the reflection must be in opposite senses, lest a 360° rotation be included.

Now the matrix of a counterclockwise 180° rotation of spinors about an axis with direction angles α, β, and γ is[12]

$$\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\cos\alpha+\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\cos\beta +\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\cos\gamma. \quad (3)$$

A clockwise 180° rotation of a spinor about its own axis will decrease ψ by 180° and will thus multiply each component in Eqs. (1) by $-i$, so that its matrix is

$$\begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}. \quad (4)$$

The reflection matrix will therefore be the product of the matrices (3) and (4), or

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\cos\alpha+\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}\cos\beta +\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cos\gamma. \quad (5)$$

To get the spin energy operator, let α, β, and γ be the direction angles of $-\mathbf{H}$ and multiply matrix (5) by $e\hbar H/2mc$. This yields for the energy operator

$$(e\hbar/2mc)\left[\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}H_x+\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}H_y +\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}H_z\right]. \quad (6)$$

The three matrices in (6) are the Pauli spin matrices and are usually denoted by σ_x, σ_y, and σ_z. It is easily verified that

$$\left.\begin{aligned} &\sigma_x^2=\sigma_y^2=\sigma_z^2=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &\sigma_x\sigma_y=-\sigma_y\sigma_x=i\sigma_z \\ &\sigma_y\sigma_z=-\sigma_z\sigma_y=i\sigma_x \\ &\sigma_z\sigma_x=-\sigma_x\sigma_z=i\sigma_y \end{aligned}\right\}. \quad (7)$$

The multiplication rules (7) are the algebraic basis for nonrelativistic spin theory and are taken as the starting point in most presentations of the subject. The Pauli matrices are identical with the three units of Macfarlane's "hyperbolic quaternions,"[13] invented about 1896. Ordinary quaternions would not quite fit the spin problem; they would meet all requirements except that they would make the energy eigenvalues come out imaginary, the reason being that the three quaternion units are 180° rotations rather than reflections in an axis.

It may be noticed that our matrices σ_x, σ_y, and σ_z are not the same as those originally used by Pauli.[14] This discrepancy is merely a matter of convention, for if we had started by defining the components of a spinor by the equations

$$\begin{aligned} a_1 &= (2a)^{\frac{1}{2}}\sin\tfrac{1}{2}\theta e^{i(\psi-\varphi)/2} \\ a_2 &= -(2a)^{\frac{1}{2}}\cos\tfrac{1}{2}\theta e^{i(\psi+\varphi)/2} \end{aligned} \quad (8)$$

instead of by Eqs. (1) (a procedure that would evidently be entirely legitimate), we would have arrived at precisely Pauli's matrices. Some writers use matrices that differ from both Pauli's and ours.[15] We have used the definitions (1) rather than (8) in order to remain in exact agreement with reference 2.

Geometrically, σ_x represents a reflection of spinors in the negative x-axis,[16] and similarly for

[12] Reference 2, Sec. 5.

[13] *Encyclopädie der mathematischen Wissenschaften* (Teubner, Leipzig, 1931), **3**, 1, 2, p. 1423.

[14] Reference 6, p. 187.

[15] E. Wigner, *Gruppentheorie und ihre Anwendung* (Vieweg, Braunschweig, 1931), p. 169; E. C. Kemble, *The Fundamental Principles of Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1937), p. 516.

[16] Hill and Landshoff state (reference 1) in their Sec. 9 that each σ represents a reflection in one of the coordinate *planes*. In the writer's opinion this is incorrect. In fact, there is no linear transformation at all that, when applied to the components of spinors, will reflect the spinors in a plane. This can be proved by investigating all the geometric operations that can be accomplished by the various linear transformations of spinors; it will be found that reflections in planes are not among them. For the details of the investigation see reference 3, Sec. 3.

It may also be pointed out (although this has no direct

σ_y and σ_z. The matrix $(\hbar/2)\sigma_x$ is usually denoted by s_x, and similarly for the other two matrices. Physically, s_x, s_y, and s_z are considered to be the quantum operators corresponding to the x, y, and z components of the spin angular momentum mentioned in the first hypothesis. The justification for this is that the classical formula for the energy of a magnetic dipole of magnetic moment $\mathbf{p}$ in a magnetic field $\mathbf{H}$,

$$-(p_xH_x+p_yH_y+p_zH_z), \tag{9}$$

agrees with the quantum energy operator (6) if p_x is identified with $-(e\hbar/2mc)\sigma_x$, and this requires, by the first two hypotheses, that the x component of the spin angular momentum shall be $(\hbar/2)\sigma_x$. [The justification given in most treatments of spin theory is that s_x, s_y, and s_z satisfy the same commutation rules as the components of orbital angular momentum, as follows at once from Eqs. (7).]

6. NONSTATIONARY SPIN STATES

In ordinary quantum mechanics, any linear combination of the eigenfunctions of a system (each provided with the proper exponential time factor $e^{i\omega_n t}$) represents a nonstationary state, whose time rate of change is given by the Schrödinger equation containing the time. If the eigenfunctions and the linear combination are normalized, the square of the absolute value of the coefficient of a given eigenfunction in the linear combination is equal to the probability that the system is in the associated stationary state.

Analogously, any linear combination of the eigenspinors for the electron in a magnetic field $\mathbf{H}$, each provided with its time factor, may be considered to represent a nonstationary spin state. The linear combination is itself a spinor, since every linear combination of two spinors is a spinor, and the nonstationary state spinors are evidently neither parallel nor antiparallel to $\mathbf{H}$. If the electron has no energy except its spin energy, the dependence of a nonstationary state spinor on the time is very simple: it rotates about the axis of $\mathbf{H}$ at an angular speed of eH/mc, because it is a fixed linear combination of the two eigenspinors, which themselves rotate about $\mathbf{H}$ at this angular speed, as shown in Sec. 4.

A spinor is normalized if it is a unit spinor, i.e., if its magnitude function $(2a)^{\frac{1}{2}}$ is unity. Let the two eigenspinors for the electron in a magnetic field be unit spinors $\mathbf{a}$ and $\mathbf{b}$, and let a given nonstationary state be represented by a unit spinor $\mathbf{c}$, which will be a linear combination of $\mathbf{a}$ and $\mathbf{b}$,

$$\mathbf{c}=m\mathbf{a}+n\mathbf{b}. \tag{9}$$

Then the coefficients m and n are the components of $\mathbf{c}$ referred to the spinors $\mathbf{a}$ and $\mathbf{b}$, respectively; and also, according to the rules of quantum mechanics, $|m|^2$ and $|n|^2$ must be the probabilities of finding the electron in states $\mathbf{a}$ and $\mathbf{b}$, respectively, when it is known to be in state $\mathbf{c}$. Therefore when an electron is in a nonstationary spin state, the probability of finding it in a given stationary state is equal to the square of the absolute value of the appropriate component of the nonstationary state spinor, or to the square of the cosine of half the angle between the handles of the nonstationary state spinor and the given stationary state spinor.[17]

This conclusion can be applied to the Stern-Gerlach experiment on silver atoms in an inhomogeneous magnetic field. Let an atom be selected whose magnetic axis is parallel to the field; then let the field be turned off and let another, differently directed magnetic field be immediately turned on. What is the probability that the atom will align itself parallel to the new field? Immediately after the new field is turned on, the atom finds itself in a nonstationary state under the new conditions, and so the probability of its being found in the stationary state inquired about must be equal to the square of the cosine of half the angle between the old and the new fields. The probability of its being found antiparallel to the new field will be the square of the sine of half this angle. These results are quite well known.[18]

bearing on the present problem) that Hill and Landshoff appear to be in error, in their Sec. 9, in stating that the inversion of spinors is accomplished by the identity transformation. Surely this cannot be correct, for the inversion must interchange the directions of any two antiparallel spinors, and the identity transformation cannot do this.

[17] Reference 2, Sec. 8.

[18] See, for example, Lindsay and Margenau, reference 1, pp. 484–486, or N. F. Mott and I. N. Sneddon, *Wave Mechanics and Its Applications* (Oxford University Press, New York, 1948), p. 99.

A concept often used in quantum mechanics is the Hilbert space associated with a dynamical system. This is defined as a space whose basis elements (or basis "vectors," as the algebraists call them) are the stationary state wave functions of the system, each provided with its exponential time factor. Every element in the space that is not a basis element is a nonstationary state function, because it is a linear combination of the basis elements. The Hilbert space for an ordinary dynamical system has an infinite number of dimensions.

These ideas can be applied to the spin problem. Evidently the basis elements of the spin Hilbert space for an electron are two unit spinors parallel and antiparallel to the external magnetic field, and the other elements of the space are all other spinors. Therefore the Hilbert space for the spin is just ordinary physical space, but with spinors, rather than points or vectors, as the elements. It is a two-dimensional space, in the sense that it has just two independent basis elements.

7. SPIN AND ORBITAL MOTION

Hitherto we have considered only the spin of an electron and have paid no attention to its translational (or orbital) motion, which must, of course, be described by ordinary quantum mechanics. To get a complete description of both the spin and the orbital motion, one uses the rules for a composite dynamical system made up of two distinct systems. Here the two distinct systems are the electron in its role as an ordinary particle and the electron in its role as something having the spin properties discussed in the preceding sections. For the details the reader is referred to the literature.

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Some Concepts in Modern Physics*

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Some concepts and ideas of contemporary physics are surveyed in order that a pattern of scientific thought broader and more meaningful than that permitted by strictly technical approach may be achieved.

THE survey of the concepts and ideas of contemporary physics, which I am presenting, may not be quite from a conventional and orthodox viewpoint. Nevertheless, the ideas do not, I believe, depart from accepted scientific knowledge and theories. I have endeavored to construct a broader and more meaningful pattern of scientific thought and concepts than a narrow technical approach would permit. What may appear as obvious in terms of the mathematical symbolism, as presented by the expert, may actually be thought-provoking, if not remarkable, upon broader analysis and reflection.

THE EGOCENTRIC OBSERVER

There is an ever-present obstacle that man has to overcome in his search for knowledge and truth and in his oft times heroic attempts to understand the universe. It is the obstacle of his egocentricity. The danger is always present that there may be too much of himself in his newly acquired knowledge, too much of what is mere psychological experience, and too little of genuine knowledge of nature.

In his first attempts to acquire knowledge of nature, man had to free himself from his innate belief that natural phenomena could be explained in terms of his own whims and desires. Later, he had to overcome the incurable vagueness and anthropomorphism of his most cherished ideas, and this led him to be critical of the accepted common-sense notions of his times. Man must beware that, in his search for knowledge, he does not merely substitute a more sophisticated type of ignorance. "It is not that people don't know, that makes them ignorant," said the great American humorist, Artemus Ward, "it is what they do know that isn't so."

In early times man projected his entire self into the outside world, and regarded the inanimate objects of his environment as endowed with life and mystical experience: a volcano was a raging demon, an eclipse an evil omen, etc. The next stage in his thinking came when man succeeded in withdrawing at least part of himself from the universe. He was content with the knowledge that physical behavior followed the same principles as the workings of his own sinews and muscles, that it was patterned after his own body. Then followed Newtonian science, and the mechanical view of nature came into being. The concepts of classical physics are based on the firm ground of our sense impressions, of our common experience and perception. The concepts of force and matter are derived from the kinaesthetic sense, mass from the tactile sense, space from a combination of our visual and kinaesthetic senses, and time from the rhythmic sense which originates in the rhythmic behavior of some of our bodily processes.

However, the great achievement of Newton and the great advance in thinking, which made possible and heralded the birth of modern science, consisted in the mathematical formulation of these concepts. Mass, for instance, was first defined as resistance to touch, but this intuitive definition fails us when we speak of light as having mass, or the mass of the electron, or the variation of mass with velocity. However, in its mathematical formulations, the concept, mass, can be perceived in such diverse phenomena as gravitation, mechanical inertia, light, and electricity.

Here I should pause to emphasize that the advances in thinking, which I am describing, are advances in scientific thinking, not in popular

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thinking. Even at the present time, the heavens are more popularly viewed from the point of view of the astrologer, rather than that of the astronomer. And one will have no difficulty in finding examples of the infantile thinking of early man in our social behavior and politics.

As long as we deal with the middle-sized world of our everyday experience—the world of averages—our thinking in terms of classical concepts remains perfectly valid and eminently successful in giving us an understanding and helping us to correlate natural phenomena at that level of experience. But when our range of experience was extended to include the great macrocosm of astrophysics and the infinite microcosm of quantum physics, we got into difficulties.

The nature of a science is reflected in its concepts, and, paradoxically enough, from the advance of science comes the knowledge that reveals the limitations of these very concepts. Not, mind you, the invalidity of the concepts. *Scientific advance does not negate the old concepts and theories, but defines their range of applicability.* Scientific advance is always toward greater objectivity and truth. Both relativity and quantum theory, but especially quantum theory, since it deals with the fundamental building blocks and processes of our universe, may be characterized as exhibiting the subjective aspect of definition and concept. Relativity has exhibited the limitations of the concept of simultaneity when applied to two events occurring at different places. Because the velocity of light is a limiting maximum velocity, the present may last longer than an instant. Its duration depends on the position of the observer who decides between past, present, and future, and the location of the events whose course in time he is investigating. Quantum theory has shown the limitations of the concepts of particle and wave in interactions where the quantum of action cannot be ignored. This circumstance arises because the concept of exact position and exact momentum fails when the quantum of action h has to be taken into account.

However, the fundamental nature of the classical concepts should always be kept in mind. In order to communicate our experimental findings and tell others what we have done, we must use classical concepts which stem from our every-day experience and perception of the outside world. The classical concepts have their roots in our language which is adapted to express our experience, and explanation and communication must be in terms of these concepts. Besides, all apparatus must have macroscopic features, and in the last analysis, all observation ultimately reduces to sense impressions.

The observer was officially recognized in physics, for the first time, by relativity theory. Armed with clocks and measuring rods, he served to illustrate in a concrete and picturesque manner its novel ideas. But neither he nor his apparatus interfered with the course of the phenomena, and the classical concept of observation, which implies a sharp and precisely defined separation between the observer and the object under observation, remained strictly valid. For this reason, relativity may be considered a classical theory having accepted the *static* concept of physical reality assumed by classical concepts. The classical concepts of field and particle, denoting separate objective elements of reality having an existence independent of observation, are completely valid in relativity theory.

One of the great accomplishments of Einstein lies in his recognition of the basic role played by the concept of invariance. It was he who first recognized the general validity of the principles of invariance and their great power. Both in the special theory of relativity and in the general theory of relativity, Einstein derived laws of nature from mathematical principles of invariance. Probably the most important principle of invariance is the one from which is deduced that the laws of nature are the same for all observers, regardless of the observer's location (he may be on the earth or on any other habitable planet) and are independent of the observer's motion, as well as independent of whatever system of reference he chooses for his measurements. In a sense, the idea of invariance may be looked upon as an objective test of physical reality. By what right, for instance, can one believe in the existence of the electron, this half-born, in many cases, unidentifiable entity, possessing no fixed intrinsic properties except those developed through interaction? Only because it represents, so to speak, an invariant of observation. No matter under what experimental conditions or circumstances, the entity we call the electron will always upon

measurement exhibit a definite mass, charge, and spin.

MATHEMATICS OF PHYSICS

We have seen how the advance of physics toward greater objectivity is coincident with the greater abstractness of its subject matter. For this reason, we should not look upon mathematics as merely something that is applied to physics, and used as a tool for calculation. It is more correct to view it as the essence of physics. How can we cope with such an ethereal, shadowy, nonlocalizable thing as the electron, except by mathematics? And there have been times when mathematics was ahead of our own understanding. It is of interest to note here that the fundamental equation of quantum mechanics, the wave equation, was discovered before its meaning was known. Schroedinger, in a lecture at Columbia University in 1926, said that he knew that his equation was mathematically correct, but he was not sure what it meant. And, in fact, only a little over a year ago there appeared a series of papers in the *Physical Review* by David Bohm, challenging the now accepted interpretation of the wave equation, and viewing the wave field as a real field exerting a force on a particle.

The mathematization of physics has become so complete that it sometimes appears that we don't have to know what we are talking about, as long as our mathematics is correct and yields results in agreement with our experimental findings.

"The only object of theoretical physics is to calculate results that can be compared with experiment, and it is quite unnecessary that any satisfying description of the whole course of the phenomena should be given," wrote Dirac in his younger days. We have since learned that this holds only in the early primitive stages of a theory. (I may add here that the Dirac quotation appeared in the first, 1930, edition of his book. I have not found it in the current edition.) Scientific progress demands a more complete theoretical understanding; and it is this understanding that is worthwhile interpreting for the nonspecialist, the scientists in other fields, and the scholar. The object of theoretical physics is not merely "to calculate results that can be compared with experiment." It is to discover the laws of nature. A competent and good theory, consequently, must have more body to it than a mere calculating scheme. The shortcomings of Heisenberg's S matrix approach lies just in the above fact.

One must not lose sight of the great humanistic value of physics. It is much more than a mere calculating game. If the vision of the artist and writer aspires to the impersonality of truth, the detachment and objectivity of the scientist aspires to getting at the suprapersonal truth free from all our deep-grained prejudices and our common misconceptions. In modern physics we have the accomplishments of man's mind at its best—when it is free to reason—and man here achieves his most characteristic, if unpopular, virtue, disinterested thought. In order to perceive and appreciate these accomplishments of modern theoretical physics, they must enter into the very texture of our thought, become part of it, and not be confined to their mathematical symbolism. It is only in terms of our language that the *meaning* of the new physical theories can be brought to light, and an awareness of their achievements brought into being; an awareness that will be of humanistic value, for it will embrace the WHOLE man, not only the mathematician in him.

Let us take another look and see why modern physics takes an abstract view of the electron, why it can grasp the electron, so to speak, only in mathematical and abstract terms. Can one expect the electron to have any sense properties, such as heat, color, smell? Obviously not, for the origin of color, smell, and heat lies in the movement of electrons. If electrons themselves possessed these sense qualities, one would no longer have a reason for the existence of these properties. And this reasoning also applies to assigning geometrical properties to the electron, such as, occupying space and position. The uncertainty principle, of course, denies the electron exact position, but generally speaking, it makes no sense to visualize any three-dimensional structure of an indivisible particle. From general considerations, one may say summarily, that the substance of substance cannot be substance, otherwise one could not explain the origin of substance and its properties.

EFFECT OF THE MEASURING APPARATUS

How, then, does the electron manifest itself as a physical entity? By interaction, is the answer given by quantum mechanics. The physical properties of free isolated electrons are but latent, and can be developed and brought into being only by interaction. Whether the electron's behavior is particle- or wave-like[1] is determined by the type of system or apparatus with which it interacts. For this reason, the apparatus has a unique dynamical role in quantum mechanics. It is not the passive instrument or system of classical physics, but constitutes the experimental conditions under which the phenomena appear. Thus, the concept of observation has not the unambiguous meaning it has in the classical domain. The concept of observation implies fixed static elements of reality. It implies that what one observes has independent existence, independent of our act of observation, and that it existed in the same form before the act of observation. In short, the concept of observation implies that there exists something that can be observed. In the quantum domain, however, there are no fixed intrinsic properties of the electron, that one can observe. The physical properties of the electron should be considered not as belonging to the electron, but rather as arising out of the interaction of the electron with the system representing our measuring apparatus. Quantum theory considers the system representing our apparatus with the atomic entity under observation as together forming a *whole*. On the atomic plane, physical reality involves interaction, interaction between the systems giving rise to the phenomena. Physical reality is thus not identified with isolated elementary systems, such as are the elementary particles, but with quantum wholes. In fact, because of the coupling of the indivisible quantum of action, or rather, interaction, *all* the systems in interaction on the quantum level are so intimately related, as strictly speaking, to constitute a WHOLE. This wholeness of the universe on the microscopic scale, which is a quantum property, has far-reaching implications. Quantum field theory does not consider even the vacuum, or empty space, as apart from the rest of the universe. Because the universe contains matter and radiation fields, empty, or vacuous, space is thereby modified. It exhibits physical properties known as the vacuum fluctuation phenomena of field, charge, and current with respect to these fields. The average values of the fluctuations will increase as the space-time region defined by the interaction or by the measurement of the fluctuations decreases. These properties should be viewed not as belonging to the vacuum, or to empty space, but as arising out of its interaction with matter and radiation fields. As in the case of the electron, explained above, it is interaction which bestows upon empty space its substance-like properties. But the interaction of empty space with matter and radiation fields is effective only over small regions of space. For larger regions, much greater than the Compton wavelength of the electron (h/mc) the effects of the electron field become negligible, are finally averaged out, and thus have no influence on our large-scale apparatus. The interaction in the small space-time regions are dominated by quantum laws and the uncertainty principle, which they embody. Because of this, any interaction that defines a small region of space, will cause zero-point oscillations of the electromagnetic field in vacuo and zero-point fluctuations of electric charge and current. Briefly, the phenomena arise as follows: the electromagnetic field is quantized by decomposing it into its Fourier components. These monochromatic waves may be considered as dynamically equivalent to a set of harmonic oscillators, and the quantization of the field then amounts to the quantization of these monochromatic waves, or oscillators. Thus, each oscillator may take on only a quantized amount of energy $h\nu$. The formula giving the energy of the field, $E=(n+\frac{1}{2})h\nu$ yields an energy $E=h\nu$ even when $n=0$, that is, even when there are no quanta present in the field. This zero-point energy implies that the electromagnetic field, even in a vacuum, will undergo oscillations, and, since in any given region of space there will be an infinite number of monochromatic waves with decreasing wavelengths adaptable to the dimensions of the region, the theory yields, FORMALLY, an infinite amount of energy for a given region of space. Because of the zero-point oscillations of the electromagnetic field, the

[1] It is significant that the correct use of the fundamental classical concepts of particle and wave is as adjectives, not nouns.

average value of the electric and magnetic field strengths, when measured over a space-time extension of the order of the electron Compton wavelength will not be zero, as one would expect for empty space, but will fluctuate upon repeated measurements. The fluctuations will become greater as the space-time region defined by these measurements become smaller. Because of the possibility of the creation of electron pairs by these electromagnetic field oscillations, (it is remembered that the electromagnetic field is here considered a system of quantized photons) there will ensue electric charge and current fluctuations that will also become greater and greater as the space-time region of measurement becomes smaller and smaller. The theory also permits an interaction of a charge, electron or proton, with the latent electrons (negative energy electrons) of space, causing a displacement of the electrons, and giving rise to what is called polarization of the vacuum. The fluctuation phenomena were not taken seriously until their physical consequence received confirmation in the now famous Lamb shift which was correctly predicted by the Schwinger theory. Recent calculations involving terms of higher order in the interaction would seem to indicate the existence of the vacuum polarization effect, as well. It is of interest to learn from a general point of view why the Dirac theory could not predict the Lamb shift.

The Dirac theory has done more to free us from acceptable and respectable common sense notions and the limitations imposed on us by such notions as probably any ordered collection of mathematical and linguistic symbols have ever accomplished. It has literally forced upon us the concept of negative energy states, the most unphysical concept in the history of physics. It is only recently that physicists have learned enough and have gained the wisdom to abandon their attempt to do away with negative energy states. It has given the electron a spin, but again, not something that can be directly and classically envisioned, like the spin of a top. On the contrary, it arises out of the *Zitterbewegung* of the relativistic electron, an unobservable trembling circular motion. It has predicted the creation and annihilation of the elementary particle, and destroyed our belief in their permanence. And yet, despite the radical nature of Dirac's theory, it failed to predict the Lamb shift. And ironically enough, the reason for its failure was because of a conservative element in the theory. It accepted the classical static point electron. Schwinger has modified the Dirac equation by cutting this umbilical cord to classical physics. He has replaced the classical point electron, static and structureless, with an electron that is an interacting system.

Consider this self-interacting electron in empty space. It is continuously undergoing a sequence of interacting processes that consist of the continual emission and absorption of virtual light quanta. The emission and absorption of these light quanta are virtual, because the time of emission and absorption is too short for observation. (They are confined within a region of the electron Compton wavelength.) These light quanta have an energy range which theoretically goes to infinity, and form parts of the field which the electron carries around with it. If energies only up to mc^2 are considered, the rest mass of the electron, the effect gives rise to the electromagnetic part of the electron's mass. However, there are terms in the expression for the self-energy, which do not have the form of the mass terms, as evinced by their relativistic transformation properties. They arise out of the interaction of the electron with those electromagnetic field fluctuations consisting of lower frequencies. The oscillation of the electron in a hydrogen atom, due to these terms, changes the average value of the potential energy of the Coulomb field between the proton and the electron by a small amount, causing the well-known Lamb shift of the hydrogen S levels (the intensity of the electron wave function at the nucleus vanishes for all states except the S states.)

STRUCTURE OF THE ELECTRON

The self-energy phenomena giving rise to the electromagnetic part of the mass of the electron as well as the other self-interaction giving rise to part of its charge involve the structure of the electron, for, in order to keep the mass and charge finite, a limiting energy or frequency of these interaction processes had to be introduced. This means introducing a radius of the electron $a=c/v$. The infinities, of course, still remain and are related in some way to the structure of the

electron. They are an indication that the free elementary particle considered in isolation is an unwarranted abstraction from physical reality. The elementary particles are not absolute but are interacting systems related either directly or indirectly to all the other elementary particles. These infinities which are of a basic nature show that the electromagnetic theory is in a sense an open one. That is to say, as we consider smaller and smaller regions of space, we shall find that electromagnetic phenomena do not exist by themselves but are connected with other types of phenomena. This will involve other matter and radiation fields and the creation of different particles other than electrons and photons.

This type of electron structure cannot, of course, be visualized in space-time, so that, when one speaks of the structure of the electron, one does not mean some space-time model, but rather a purely quantum conception. Structure, up to now, was considered an unphysical concept, for it had no meaning to speak of the electron's structure in the classical sense. But the advance of physics did present the physicists with problems involving the structure of the electron, although it was of a kind they could not have imagined, since it could not be envisaged by any *a priori* space-time model. Because advances in contemporary theoretical physics are mathematical and abstract in nature, the body of abstract thought which marks those advances must first be discovered. They can then be cast in terms of our classical concepts and language, in order to assume physical features and form, not the other way around. This is why it is not possible to predict, on the basis of classical knowledge what the future concepts of physics will be, or what new meaning the classical concepts will have in future theories. Generally speaking, it is not possible to predict, from present knowledge and contemporary fashions of thought and prejudices, what form physical theories will have. However, this is a small price to pay for objective knowledge. We must reconcile ourselves to the role of scientists seeking after the truth, not prophets, even those speaking with scientific authority, who know what form the truth will take.

We have noted the omnipresence of interaction: the interactions associated with our act of observation, which give the elementary particles their properties; and then, even the self-interactions associated with unobservable phenomena, but which are involved in the structure and existence of the elementary particle, and which have observable consequences. The classical concepts of the static point particle or the free field are but theoretical idealizations. They are, what I have called elsewhere, INCOMPLETE SYSTEMS. Einstein believes that the statistical character of quantum theory arises out of its incompleteness. On the contrary, it is classical theory which is incomplete, for reality can be grasped only by the trinity—particle, field, and interaction, not by any fixed static elements. Change, in our universe, is indeed fundamental.

The world picture given to us by quantum physics is radically different from the classical world picture. It is believed by Dirac and others that some of the shortcomings of quantum theory may be traced to working from a wrong classical theory. Of course, one must retain the classical concepts which are fundamental and deeply rooted in our common experience and language. However, since it is the quantum theory which is primary and fundamental, one should endeavor to get a classical theory fashioned out of the knowledge gained from quantum theory, and whose world picture may not be at such total variance with the quantum picture. This, in the author's opinion, is the significance of Dirac's new classical electron theory. It is too early to tell at the present time whether Dirac's theory will have any permanent value. But, right or wrong, it is an attempt to recognize the coexistence of particle, field, and interaction, all on an equal dynamic footing. In Dirac's theory, the electron is not considered apart from its electromagnetic field.

We have seen how, in quantum theory, empty space exhibits dynamic or interacting properties; in particular, that it exhibits unobservable fluctuation properties with respect to the quantized electromagnetic field. And so, in Dirac's theory, space takes on unobservable velocity properties with respect to the electromagnetic field. Space, in Dirac's theory, to use Professor Rosenfeld's word "flows," and despite the fact that its flow is unobservable, space becomes more of a thing than it was before. We thus honor

it by giving it a name, and the ether is rediscovered in classical physics. It is the great "unobservable" of Dirac's classical electron theory.

One of the consequences of the existence of a velocity field in empty space is that, even in the vacuum, under which condition one may consider the existence of a free electromagnetic field, or free charges, the velocity field prevents all the field quantities from becoming zero. Thus, Dirac's theory permits the appearance of charge in the vacuum without destroying the vacuous character of the region. One may have a free electromagnetic field in the vacuum, and, at the same time, admit the possibility of the appearance of a small charge. But this condition is true only in the zeroth approximation. Rigorously, in the higher approximations, one must consider the "vacuum interaction" between the free electromagnetic field and the small charge. This implies that the fundamental concepts of free particle and field existing by themselves, separate from their interaction with each other, are only approximate concepts, even in the classical domain.

The omnipresence of interaction throws new light on the physical meaning of Dirac's ether. The abstraction of interaction apart from the presence of all matter and field, may be identified with the ether concept. This abstraction is just as legitimate as that of particle or field, which, after all, do not exist apart from their interaction with each other. Employing Dirac's terminology, one may say that all three—particle, field, and ether (interaction)—may be considered as "theoretical idealizations."

UNOBSERVABLE PROCESSES

Now, some reflections on the physical importance of unobservable processes. In the early days of quantum theory, it was believed that one could have a theory consisting only of observable concepts. Now we know that such a program is not possible. Without the unobservable concepts and processes, there could be no quantum field theory. *The unobservable processes in quantum theory have direct physical consequences.* To begin, consider the concept of intermediate states. The transitions to these states do not last long enough to be measurable, and yet they trigger some of the important and fundamental transitions in quantum field theory. This is because there are many important processes for which the theory does not permit direct transitions. These are processes for which there exists no electric moment, classically speaking, and are thereby forbidden. However, the theory permits the transition to be made by first going to an intermediate state, and then to the final state. The transition to the intermediate state need not generally obey the energy conservation law, but, since the transition is unobservable, there ensues no difficulty with the conservation law. Another group of unobservable processes fundamental for the theory is the matter and field fluctuations, which we spoke about before. Despite the fact that they are fundamentally unobservable, they have important measurable consequences, such as in the Lamb shift, and in the anomalous magnetic moment of the electron. Then there are the unobservable processes which themselves constitute the phenomenon, and under such a category come nuclear forces. It is generally accepted that nuclear forces arise from the exchange of mesons between nucleons. These mesons, or pions, as they are now called, are about 276 times as heavy as the electron and are continually being emitted and reabsorbed by the nucleon. This process may be pictured as the nucleons throwing one another's pions around. Roughly speaking, the attraction of the nucleons for one another's pions is what keeps the nucleus together. The distance the pions are thrown, before being intercepted, is equal to the range of nuclear forces. And yet, simple as the picture is, how completely and radically different it is from any classical model. In fact, it is beyond the competence of any classical theory to grasp or conceive, for it involves an outstanding violation of the energy conservation law from the classical point of view. Not only do the nucleons steal one another's pions, but the pions themselves were never paid for by any of the nucleons. Their creation involves an expenditure of about 138 million volts for each. Strangely enough, it is the uncertainty principle which saves the conservation law here, as it oft times has done in unobservable processes. The distance the pions are thrown about is so short, that the phenomenon does not last long enough to be observed.

Nuclear forces may involve heavier and heavier mesons. However, the heavier the mesons

the shorter the range of the field, which range is equal to its Compton wavelength. Consequently, the effects of these fields are confined to shorter and shorter distances as the quanta of the field become heavier and heavier. If there is an infinite energy residing in space, it resides fortunately, in an infinitesimal volume of space, and consequently, it cannot be detected or made use of. Conservation laws come only with the observable effects, after the mass and charge of the particle have been developed, that is, after the self-energy effects. More generally speaking, one may say that the conservation laws *assume* the existence of the particle; they cannot explain its existence. With the exception of the electromagnetic field where quanta have zero mass, all the fields in nature are short-range fields, their range being equal to the Compton wavelength of their associated particles. These fields, consequently, have no effect on physical objects of classical size and cannot be directly detected by our measuring instruments.

Nevertheless, their existence has a revolutionary implication for human thought. One of the fundamental assumptions of classic science and philosophic thought is that, in the absence of matter, all space is alike and has the same geometrical properties. The existence of these short-range fields, coupled with the vacuum fluctuations of these fields, implies that even this assumption is no longer tenable, for it has the consequence that small regions of space of extension smaller than the electron Compton wavelength are different from large macroscopic regions of space. These small regions of space have new and radically different properties from the properties of macroscopic space. They are of a physical dynamical nature, rather than of a geometrical static nature. Moreover, as these microscopic regions of space become smaller and smaller, they differ from one another, due to the emergence of new fields with heavier quanta.

Summarily, one may say that the strength of a quantum field theory, like that of a living organism, is conditioned by its unobservable features. Whether a quantum field theory is in need of repair or revision, will oft times depend on the unobservables—whether they are well-ordered and understood, for there are physical phenomena that rest on these unobservables. The observables of a theory are comparatively easy to handle—the test and strength of a field theory is what it does with the unobservables, or what the unobservables do with it. The unobservable and divergent features of field theories may not be something temporary. We may be canceling or juggling infinities for a long time to come. It is possible that we shall get rid of them only when we have a complete and final theory of the physical universe—when we have a theory that does not ignore the intimate relatedness between the elementary constituents of the universe. It is possible that the occurence of the infinities and divergences which are allied to the unobservables is the price we have to pay for the finiteness of physics in an infinitely complex world, or, rather, it is the price we pay for having a science of physics, for it is human intelligence which is finite. After we have understood and become better acquainted with the unobservables and their divergent properties, we may consider them to be no more of a shortcoming to physical theory and thought than the uncertainty principle. It is remembered that in the early days not a few physicists thought that the uncertainty principle was but a manifestation of the incompleteness of quantum theory.

LIMITATIONS OF OBSERVATION

In modern quantum field theories, the chain of inference from observation to theory is getting longer and longer. This is probably one of the least expected results of modern theoretical research. The laws of nature, contrary to the opinions of many leading physicists, are more than statements comprising the properties of our measuring appliances and our apparatus. Nature cannot be encompassed from so subjective a standpoint, even if that standpoint be an experimental one. The phenomena which our experiments give rise to are not nature itself. Our large-scale apparatus cannot penetrate to the fundamental level of the short-range quantum fields whose quanta are the elementary particles of nature. Events on this level cannot be directly observed, due to a theoretical limitation inherent in the very process of observation, as a result of the uncertainty principle. Here one has another good reason for believing that the quantum laws are true basic laws of nature, for they

imply that not even the man of science, with his powerful apparatus of observation, is the center of the universe. The quantum laws do not permit his omnipercipience. The unobservable phenomena connected with the behavior of the short-range quantum fields are not, as we have seen, a chimera, for they influence and trigger certain events on the observable level. However, I am not implying limitations to man's search for knowledge, only limitations to his most cherished ideas and concepts. As long as nature can be comprehended by abstract rigorous thought and reasoning, if not by imagery and *a priori* conceptions, man's search for knowledge and truth will be fruitful. Neither experiment nor theory should outstrip each other, but go hand-in-hand. On the complex and fundamental levels of nature, each must guide the other.

What a far cry is this situation from that of 1926, when quantum theory was first discovered. At that time, it was generally believed, as was previously stated, that the new theory dealt exclusively with observable quantities. The belief arose from the fact that one of the fundamental differences between the new quantum theory and the old Bohr theory was the omission of the physical quantities, such as the orbital motions of the individual electrons in the atom, which are not observable. The inclusion of the orbital trajectories of the individual electrons would have been in direct violation with the new theory, as expressed by the uncertainty relation. It was therefore reasoned incorrectly that the laws of nature, as expressed in the laws of quantum mechanics, do not tolerate any inclusion of physically unobservable quantities. The justification of this belief on a broad, general basis was not difficult, because it fitted so nicely with the positivistic philosophy.

No philosophical principle, no matter how reasonable it may appear, can encompass all that will be known about nature. For that reason, one should not try to predict or dictate the future course of physical theory on some philosophical principle, no matter how sound this principle may appear to be. The dictum that every physical concept and quantity should have an operational meaning, is only an expression of an attitude engendered by the critical examination of our classical concepts initiated by relativity theory and the early appraisals of quantum mechanics. It was useful in the early days, when it helped one to understand some phases of these great discoveries. We have learned, since, that physics is not so simple, and cannot be encompassed by any great guiding principle. It remained for quantum mechanics to shake our belief in the fundamental simplicity of physics. Relativity has demonstrated that *space and time are inseparable*. Quantum theory has made an even more revolutionary finding, as has been emphasized throughout this paper. It has shown that, on the atomic plane, there exist no separate isolated elements of reality, such as, particle or field; particle, field, and interaction are, strictly speaking, inseparable. The three together form a whole. When the interaction is strong, that is, when the coupling between particle and field may not be considered small, and, in the nuclear domain it is never small, then the underlying assumption for our belief in the simplicity of the physical world fails. The assumption is that one can legitimately isolate simple laws, particles, and fields in natural phenomena, and later recombine them to predict the whole phenomenon. For instance, in the study of the trajectories of projectiles, one can legitimately isolate the gravitational force, as expressed in the gravitational law, from other physical effects, such as, air resistance, etc., and study each effect separately. Then one can recombine them to predict correctly the path of the projectile. This type of simple procedure, wherein the physical variables are "additive" and the coupling parameter between the interacting systems may be permitted to go to zero, assumes the strict validity of the concept of the free, individual particle—the simple mass point. This assumption may not always be warranted in the domain of quantum field theory. In fact, when the coupling between the particles and field is strong, the separation between the two is no longer possible. The interacting systems cannot be considered free, and the interaction treated as a perturbation. The limitations, even the failure of perturbation theory and the perturbation approach, may ultimately spell the failure of simplicity as a guide post to future theoretical research in high energy physics and field theory. The classical world of Newton and Einstein, which can account for the motion of the

projectile in terms of the geometry of the space-time continuum, gives way to a world whose chief aspect is CHANGE, a world which cannot be caught on a gigantic four-dimensional canvas. Change regards interaction not as a mere perturbation, but as a fundamental property of the universe. As we come to smaller and smaller regions of space, the failure of our simple laws and concepts become more pronounced and accentuated. This is the reason why, in nuclear physics, every simple solution, so far, has brought forth but new complexities.

ROLE OF STATISTICS

There is another feature of quantum mechanics which I should like to talk about. It is a feature which completely defies any classical explanation or analogy: the role of the statistics, and of the exclusion principle. The exclusion principle, as is well known, is a mathematical way of formulating the simple fact of the impenetrability of matter. Mathematical formulations and scientific explanations of basic intuitive conceptions and simple physical "facts" are more subtle and profound than the sense impression of these facts, vivid as they may be, would lead one to believe.

Electrons, protons, and neutrons, which comprise matter, all obey the exclusion principle and the Fermi-Dirac statistics. The Fermi-Dirac statistics is the type of statistics applicable to particles which obey the exclusion principle. The import of the exclusion principle and its statistics is that no two electrons can occupy the same quantum state (designated by the four quantum numbers: n, the principal quantum number; l, the azimuthal quantum number; m_l, and m_s, the orientation of orbital angular momentum and spin, respectively). Also, there can be only one electron with the same space coordinates and spin. As a result, two electrons tend to keep away from each other. The probability that they would be found closer and closer becomes less and less, and the probability is zero that two electrons of the same spin would be found to occupy the same space at the same time. Since matter is not solid chunks of anything, this may be said to express the impenetrability of matter from the quantum point of view. It is remarkable that it is not any *force* that keeps the particles away, but the fact that they obey a certain type of *statistics*. Since this effect is not due to a force at all, it can manifest itself instantaneously. For this reason, there are instances when it seems to control the operation of forces.

An example of this is in the suppression of the strong interaction between nucleons, when they are in their lowest energy states. This may be viewed as a consequence of the Pauli exclusion principle which does not permit energy transfers between such nucleons, because all the low-energy states which can be reached by such transfers are already occupied. It is the properties of these nuclear ground states that support the shell model of the nucleus. On the other hand, the shell model is in direct contradiction with the compound nucleus model, which implies the existence of strong attractive forces between nucleons, ample evidence for which has been gathered from a study of nuclear reactions. The manifestation of these forces in nuclear reactions is in accord with the Pauli principle which permits strong energy transfers between nucleons in highly excited nuclei, because the states in the high-energy regions are not all occupied. Thus, the Pauli principle helps one to understand, in a qualitative way, why a model based on strong forces and another based on weak forces are not necessarily in disagreement.

In the nuclear domain, simple pictures and models, such as the shell model, should not lead one to expect that simple solutions necessarily exist for nuclear problems. The one-body potential which the nuclear shell model suggests, is likely only a smoothed-out, averaged-in-time version of an infinitely more complicated interaction.

MACROSCOPIC PHENOMENON BASED UPON QUANTUM LAWS

Discussions of quantum-mechanical laws have generally emphasized their statistical aspects, and often neglected to point out the important and fundamental fact that these laws *do govern*. They impose, essentially, a different kind of behavior, often strange and bizarre, from that imposed by classical laws. We have learned to accept the strange behavior of the fundamental units of matter without fully realizing its implications. In our macroscopic world this behavior is blurred by thermal motions, and averaged out

by the infinitely great number of atoms in a macroscopic piece of matter. After all, the atomic and subatomic worlds are quite remote from the psychologically real world of our everyday experience. However, in the existence of the remarkable superfluid properties of helium near absolute zero, it seems that one comes across, for the first time, macroscopic phenomena in accordance with quantum laws. One can, for the first time, SEE the utter queerness of quantum-mechanical behavior. It has become real to us.

The transition temperature of 2.19°K is the critical temperature below which liquid helium exhibits quantum-mechanical behavior, and above which it behaves like any other normal liquid. The condensed phase, known as the superfluid, sets in at this transition temperature, and is designated as liquid helium II. Liquid helium II consists of a mixture of the normal fluid whose atoms are in excited states, and the quantum fluid or superfluid whose atoms are all in the lowest quantum energy state. At absolute zero, liquid helium II would be all superfluid, as all the atoms would be in the lowest energy states. It is assumed that the normal liquid and the superfluid interact very little with each other.

The unexpected begins at the very birth of this remarkable fluid, for it should have been a solid instead of a liquid. Ordinarily, when a liquid is cooled sufficiently, it solidifies. The superfluid exhibits no viscosity, no friction in its flow, hence its name. Because of its frictionless flow, it can penetrate cracks and spaces too small for even a gas to go through them. Experience, as well as thermodynamical principles, leads one to expect heat to flow from a higher temperature to a lower temperature. The phenomenon is so familiar, that we deem it a natural phenomenon. But the superfluid can go from a lower temperature to a higher temperature, and, in doing so, can creep up along the walls of the containing vessel. It can convert heat into work with practically 100 percent efficiency, a feat which no machine living or nonliving can do. As a heat conductor, this remarkable fluid is about a million times better than the best metal conductor.

This remarkable behavior of liquid helium arises from the circumstance that it obeys the Bose-Einstein statistics (nuclei with even number of elementary particles obey the Bose-Einstein statistics, those with an odd number, the Fermi-Dirac statistics). The Bose-Einstein statistics permits any number of particles in one state, consequently, at absolute zero, the state of lowest energy, one should expect to find all the particles in one state. Now, while molecules in one state cannot be distinguished from one another, they can, at least, be counted. And yet, the Bose-Einstein statistics destroys even this last vestige of individuality. According to this statistics, not even different permutations of the molecules should be counted. That is, if molecule A, say, is to the left of molecule B in one case, and to the right of molecule B in another case, one should not count them as two separate cases, but rather as one single case. This complete loss of molecular individuality implies, according to Einstein, a statistical tendency of all the molecules to have the same velocity, giving rise to a condensation of all the molecules in the lowest energy state.

Owing to this sharp definition of momentum in the lowest energy state, there will be a correspondingly large uncertainty in the position of the molecules, an uncertainty of the order of, or larger than, the distance between them. One cannot, therefore, describe the system by means of wave packets representing the different helium atoms, but rather by a common wave function representing the whole superfluid spread over the entire volume. Consequently, in this system, there is no energy transfer by the usual well-known collision process, for, in a sense, there are no dynamical parts to the system. One has a perfectly frictionless flowing fluid. The condensation of the superfluid atoms in the lowest energy state and their complete loss of identity follows, as shown above, from the Bose-Einstein statistics. In accord with this aspect, the superfluid may be said to behave as if it were a giant supermolecule. In my opinion, what is even more remarkable than the remarkable properties of liquid helium, is that they are a consequence of the way of counting the helium molecules, not from any interaction, force, or physical agency of any kind whatsoever. Recent experiments support the unique role the statistics play in quantum mechanics. These experiments were made

possible when ample quantities of the rare stable isotope He^3 had been obtained. This isotope, containing an odd number of nucleons, obeys the Fermi-Dirac statistics which counts the permutations of the molecule in an entirely different way, and thus does not permit the condensation-phenomenon. It was found that He^3 near absolute zero does *not* exhibit any superfluidity. From experimental evidence, one would deduce that He^4 and He^3 are completely different elements, near absolute zero, instead of two isotopes of the same element, differing only because they obey different statistics.

Modern physics, which is reaching deep into the nature of things, is revealing the miracle of the substratum out of which the world of our senses is built. In the manner of a true natural philosophy, physics begins with the desire for knowledge and truth, and ends in wonder. Not, mind you, in the passive contemplation of wonder, but in the wonder revealed by newly discovered truths of nature, and in the miracle of existence.

Steady-State Shape of a Wire Cutting a Uniform Medium

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Equations are derived for the steady-state shape of a wire cutting a uniform medium, and some wire shapes are shown. The principal assumption is that any element of length of the wire moves into the medium at a rate proportional to the kth power of the normal force per unit length that the wire exerts on the medium.

AT the U. S. Naval Ordnance Test Station an experiment has been performed in which the hot junction of a thermocouple was kept in contact with the surface of a burning propellant. The experimental procedure was simply to have the thermocouple wire in tension against the surface of the propellant with the thermocouple junction approximately at the center of the burning surface. Above the wire there was a flame and below it the propellant.

This is one of many experiments that have been performed as part of a study of the burning process. One can obtain a temperature reading from it, and is able to measure such things as wire radius, shape, and tension, and the speed with which the wire moves into the propellant. When analyzing the results obtained from this experiment, it is of interest to know what is happening directly below the thermocouple junction. Among other things, one wishes to know the pressure that the wire exerts on the propellant. This may be calculated if the tension and curvature of the wire are known. It is the purpose of this paper to determine the curvature on the basis of certain assumptions. Experimental details, together with results and analysis of results, are being published by M. H. Hunt as a NavOrd report.

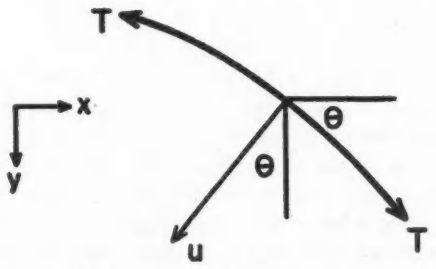

FIG. 1. Showing relationship between θ, u, and T.

Although this paper was stimulated by the experiment just described, the analysis that follows will be equally likely to be applicable to other cases of wires cutting media, as for example the regelation experiment, where a wire cuts through a block of ice.

It will be assumed that the tension in the wire is constant throughout the length under consideration, and that this tension does not change with the time. It will be assumed also that any element of length of the wire moves into the

medium at a rate proportional to the kth power of the normal force per unit length F that the wire exerts on the medium. Then

$$u = CF^k, \tag{1}$$

where u is the magnitude of the velocity of the wire in the direction normal to the surface of the medium, and C is a proportionality constant. It turns out that the value of k determines the shape of the wire. Assuming that Eq. (1) holds, we can obtain an approximate value for k from the shape of the wire. In the case of the burning propellant, for example, this is of interest because, with the aid of certain assumptions, one can estimate k from other experiments. The

TABLE I. Rectangular coordinates of the wire as calculated from Eqs. (12)–(14).

	x/r_0			Y/r_0		
θ	$k=0.5$	$k=1$	$k=2$	$k=0.5$	$k=1$	$k=2$
0°	0	0	0	0	0	0
5	0.0874	0.0873	0.0870	0.0038	0.0038	0.0038
10	0.1754	0.1745	0.1740	0.0154	0.0153	0.0153
15	0.2648	0.2618	0.2603	0.0353	0.0347	0.0344
20	0.3564	0.3491	0.3454	0.0642	0.0622	0.0612
25	0.4509	0.4363	0.4294	0.1034	0.0984	0.0960
30	0.5493	0.5236	0.5116	0.1547	0.1438	0.1388
35	0.6528	0.6109	0.5917	0.2208	0.1995	0.1899
40	0.7629	0.6981	0.6694	0.3054	0.2665	0.2495
45	0.8814	0.7854	0.7443	0.4142	0.3466	0.3182
50	1.0107	0.8727	0.8160	0.5557	0.4419	0.3965
55	1.1542	0.9599	0.8841	0.7434	0.5559	0.4853
60	1.3170	1.0472	0.9480	1.0000	0.6931	0.5858
65	1.5065	1.1345	1.0073	1.3662	0.8613	0.6998
70	1.7354	1.2217	1.0612	1.9238	1.0729	0.8304
75	2.0276	1.3090	1.1091	2.8637	1.3516	0.9825
80	2.4362	1.3963	1.1496	4.7588	1.7507	1.1666
85	3.1313	1.4835	1.1810	10.4737	2.4400	1.4096
90	∞	1.5708	1.1981	∞	∞	2.0000

quantity F may, of course, be related to the pressure P under the wire. For example, if the pressure is constant, then $F=2Pa$, where a is the radius of the wire.

If the wire has negligible bending moment, it may be shown[1] that its radius of curvature r at any point is related to the tension T and normal force per unit length F at that point by the equation

$$T = Fr. \tag{2}$$

Let θ be defined as in Fig. 1, where the lines shown are all in the plane of the paper. If s is

[1] W. D. MacMillan, *Theoretical Mechanics: Statistics and Dynamics of a Particle* (McGraw-Hill Book Company, Inc., New York, 1927), first edition, p. 87.

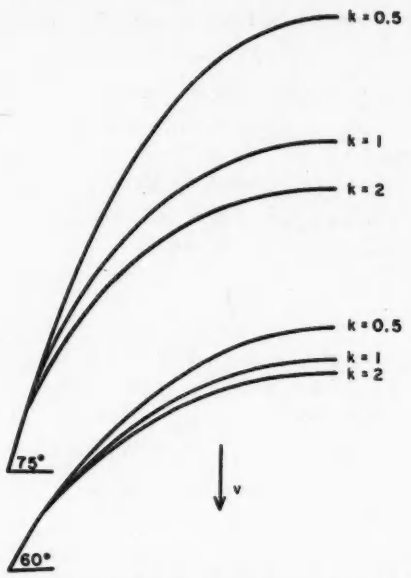

FIG. 2. Some wire shapes calculated from Eqs. (12)–(14).

distance measured along the wire at any instant, then

$$1/r = (\partial\theta/\partial s)_t = \cos\theta(\partial\theta/\partial x)_t, \tag{3}$$

where s is measured to the right in Fig. 1, and t is the time. Also,

$$u = \cos\theta(\partial y/\partial t)_x. \tag{4}$$

From Eqs. (1)–(4), one obtains

$$\cos\theta(\partial y/\partial t)_x = C[T\cos\theta(\partial\theta/\partial x)_t]^k. \tag{5}$$

As a matter of interest, note that this becomes of the same form as the standard heat-flow equation if $k=1$, and θ is small enough so that

$$\theta \approx \tan\theta = (\partial y/\partial x)_t. \tag{6}$$

It will next be assumed that the steady state has been reached so that the wire of Fig. 1 is moving downward with constant speed and shape. Call the constant speed v. That is, set

$$v = (\partial y/\partial t)_x = \text{constant}. \tag{7}$$

Then Eq. (5) becomes

$$\left(\frac{v\cos\theta}{C}\right)^{1/k} = T\cos\theta\frac{d\theta}{dx}. \tag{8}$$

Let the radius of curvature of the wire be r_0

when $\theta=0$. Using Eqs. (3) and (8), one finds

$$T=r_0(v/C)^{1/k}, \tag{9}$$

and so Eq. (8) may be written

$$dx/d\theta=r_0\cos^{1-1/k}\theta, \tag{10}$$

where

$$\tan\theta=(\partial y/\partial x)_t. \tag{11}$$

If x is set equal to zero when θ is zero, then the integrated form of Eq. (10) is

$$x=r_0\int_0^\theta \cos^{1-1/k}\theta d\theta. \tag{12}$$

When x is eliminated from Eq. (10) by means of Eq. (11), the resulting equation can be integrated to give

$$(y-y_0)(k-1)=kr_0(1-\cos^{1-1/k}\theta) \quad \text{for } k\neq 1 \tag{13}$$

$$y-y_0=r_0\ln\sec\theta \quad \text{for } k=1, \tag{14}$$

where y_0 is the value of y corresponding to $\theta=0$.

From Eqs. (1), (2), (4), (7), and (9), the radius of curvature of the wire at any point is found to be

$$r=r_0/(\cos\theta)^{1/k}. \tag{15}$$

Table I shows values of x and $Y=y-y_0$ calculated from Eqs. (12)–(14) for the cases where $k=0.5$, 1, and 2. Some wire shapes are shown in Fig. 2. These curves were obtained from the data in Table I by choosing values for r_0 such that θ would be 60° and 75°, respectively, for x equal to unity.

The only trouble involved in calculation of the values given in Table I was the determination of x/r_0 when $k=2$. For this case x can be expressed either in terms of the incomplete beta function, B_λ (0.75, 0.5), or as the difference between two incomplete elliptic integrals. There do not appear to be tables of B_λ (0.75, 0.5) as a function of λ, and so x was obtained as the difference between two elliptic integrals as follows. Set $k=2$, and

$$\cos\theta=\cos^2\phi \tag{16}$$

in Eq. (12). When this is done, x may be put in the form

$$x=r_0\sqrt{2}\left[2\int_0^\phi (1-0.5\sin^2\phi)^{\frac{1}{2}}d\phi - \int_0^\phi (1-0.5\sin^2\phi)^{-\frac{1}{2}}d\phi\right]. \tag{17}$$

The integrals in this equation are incomplete elliptic integrals of the second and first kind, respectively, and are tabulated in various places. Since the results are not obtained as a function of θ, it was necessary to interpolate to get the values shown in Table I for x/r_0 when $k=2$. Although considerable care was taken in interpolating, there is, nevertheless, a chance for error in the last significant figure given.

The author wishes to thank Mr. Maynard Hunt for suggesting this problem to him, and Dr. John W. Odle for reviewing the manuscript.

Reproduction of Prints, Drawings, and Paintings of Interest in the History of Physics

55. Titlepage from Robert Grosseteste's *Lines, Angles, and Figures*

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(Received June 16, 1953)

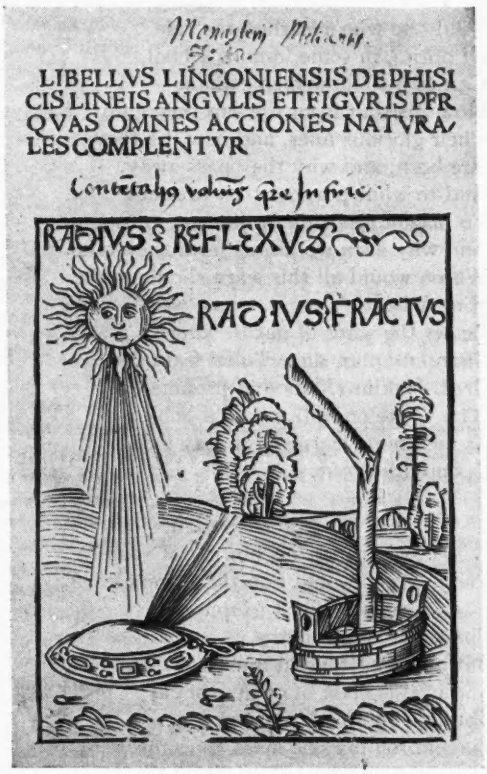
Probably no one had a greater influence upon English thought and literature during the thirteenth and fourteenth centuries than Robert Grosseteste. It is proposed to celebrate the seven-hundredth anniversary of his death by reproducing a curious woodcut, portraying the reflection and refraction of light, which forms the title page of his *Libellus de Phisicis Lineis Angulis et Figuris* (Nuremberg, 1503).

ROBERT GROSSETESTE (*ca* 1175–1253), first Chancellor of Oxford University and Bishop of Lincoln from 1235 to 1253, was one of the most original thinkers of mediaeval times. He was a mathematician, astronomer, and physicist, as well as a philosopher, linguist, churchman, statesman, and man of affairs. As GEORGE SARTON has said in his *Introduction to the History*

of Science,[1] "he was the main organizer of philosophical studies at Oxford, and his influence was strongly felt in England for a least a couple of centuries. Nor was it limited to England. His insistence on the necessity of studying Greek and of basing natural philosophy upon mathematics and experiment was extremely beneficial and far-reaching; in this he was clearly the forerunner of his most famous pupil, Roger Bacon. We may say that he influenced the whole western world, partly through his own writings, and partly through these new tendencies emphasized by Bacon and others." It would seem appropriate to celebrate the 700th anniversary of the death of such a man by reproducing in this series the woodcut title page of one of his physical papers which was published in Nuremberg in 1503.

Brief accounts of GROSSETESTE's life and work, together with more complete references are readily available in the *Dictionary of National Biography*, the 11th Edition of the *Encyclopaedia Britannica*, George Sarton's *Introduction to the History of Science*, and the *History of Magic and Experimental Science* by Lynn Thorndyke. More recent references not mentioned by these authorities are Crombie's *Robert Grosseteste and the Origins of Experimental Science*[2] and *Augustine to Galileo. The History of Science A.D. 400–1650*.[3] For the purposes of this article, however, it is only necessary to point out that GROSSETESTE was the first outstanding student of optics in the West and that he set the direction for future developments in this field. Although his chief contribution was to emphasize the value of experimental and mathematical methods, he also made important additions to positive knowledge. Thus he gave a quantitative law of refraction based, he says, upon experiment and the principle of economy—a law which, although not correct, was used until the sixteenth century to explain the action of convex lenses or burning glasses. He was also the first Latin writer to suggest using lenses to magnify small objects and bring distant objects closer. As Sarton has said, it is no disparagement of ROGER BACON to say that he owed his knowledge of the magnifying power of lenses and much other such information to his teacher.

As an illustration of the extent of GROSSETESTE's knowledge of optics, it may be worthwhile to quote from his treatise on the rainbow[4] in which he states that one of the branches of what he calls Perspective if "thoroughly known shows us how to make things very far off seem


LIBELLVS LINCONIENSIS DE PHISICIS LINEIS ANGVLIS ET FIGVRIS PER QVAS OMNES ACCIONES NATVRALES COMPLENTVR
RADIVS REFLEXVS
RADIVS FRACTVS

"The Reflection and Refraction of Rays" [Titlepage to Robert Grosseteste's *On Lines, Angles and Figures* (Nuremberg, 1503)].

very close at hand and how to make large objects which are near seem tiny and how to make distant object appear as close as we choose, so that it is possible for us to read the smallest letters at an

[1] George Sarton, *Introduction to the History of Science* (Williams and Wilkins, Baltimore, Maryland, 1947), Vol. II, pt. 2, p. 583.

[2] A. C. Crombie, *Robert Grosseteste and the Origin of Experimental Science* (Clarendon Press, Oxford, 1953).

[3] A. C. Crombie, *Augustine to Galileo. The History of Science A.D. 400–1650* (Falcon Press, London, 1952).

[4] *De iride, seu de iride et speculo.* This paper will be found in Ludwig Baur's *Die Philosophischen Werke des Robert Grosseteste*, which was published in C. Baeumker's *Beiträge sur Geschichte der Philosophie des Mittelalters*, Vol. 9 (Münster, 1912).

incredible distance, or to count sand or grain or grass or any other minute objects."[5] In this connection it is interesting to quote a few lines, which seem to paraphrase GROSSETESTE's teachings, from the continuation of the celebrated poem, *The Romance of the Rose*, written about 1270 by JEAN DE MUEN (surnamed CLOPINEL).[6]

After a magnificent description of a storm followed by a rainbow and the return of fair weather, the poet continues as follows:

"Only he who's learned the rule
Of optics, in some famous school,
Can to his fellow-men explain
How 'tis that from the sun they gain
Their glorious hues, and how 'tis they
Are born, and why they pass away,
And to what purpose they are sent
To blazon the wide firmament,
And why such wondrous form they take.
Whoso would all this learn should make
Him Aristotle's pupil, who
Better the ways of nature knew
Than any man since Tubal Cain,
Or Al-Hakim (Mahound profane),
Who wrote on optics such a book
As men most learned yet may look
Upon with profit.

And each man,
Who of the wondrous bows that span
The heavens would fain the secrets learn,
In no wise can afford to spurn
The study of geometry,
But an adept therein must be,
And of this book of optics should
Obtain a thorough masterhood;
Therein too may he learn the cause
Why mirrors, through some subtle laws,
Have power, to objects seen therein
(Atoms minute or letters thin),
To give appearance of fair size,
Though naked, unassisted eyes
Can scarce perceive them; grains of sand
Seem stones, when though these glasses scanned;
These may we count, and those may read,
From such a distance, that, indeed,
If one should tell the things he's seen,
Small credit would he win, I ween,
From him who'd seen it not, nor knew
How 'twas that such great marvels grew.
But to these matters, blind affiance
No man need give, they're proved by science."

The woodcut here reproduced appears on the title page of the *Libellus Linconiensis de Phisicis Lineis Angulis et Figuris per quas omnes acciones naturales complentus* (Nuremberg, 1503). GROSSETESTE begins this tract of eight pages by stating that a consideration of lines, angles, and figures is of the greatest usefulness because all causes of natural effects have to be expressed by means of them and it is impossible to understand natural philosophy without their use. He then argues that all action at a distance, whether by means of light or heat rays, astrological influence, mechanical or other action, is brought about by the propagation of rays of force or, as he calls it, the "multiplication of species" or "virtue." (This is one of the earliest attempts to come to grips with the problem of the nature of action at a distance, a problem that has since challenged some of our greatest minds.) Since the most convenient form in which to study this propagation of "species" is through visible light, the study of optics is of great significance for the understanding of the physical world. He makes use of two fundamental assumptions, (1) the principle of uniformity of nature, and (2) the principle of economy. "Nature operates in the shortest possible way," he says, and he uses this as an argument to support the law of reflection[7] and his own theory of refraction.

[5] Translation by Lynn Thorndyke in *A History of Magic and Experimental Science* (Columbia University Press, New York, 1923), Vol. II, p. 441.

[6] *The Romance of the Rose*, by W. Lorris and J. Clopinel [English translation by F. S. Ellis in the Temple Classics (Dent & Company, 1900 and 1927), Vol. 3, pp. 113–115].

[7] The proof from the principle of economy that the angles of incidence and refraction of light are equal was first given by Heron of Alexandria (probably first century A.D.). See Sir Thomas Heath, *A History of Greek Mathematics* (Clarendon Press, Oxford, 1921), Vol. II, p. 353.

NOTES AND DISCUSSION

An Audible Repeating Time Signal Device

T. G. BULLEN
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MANY laboratory exercises require the noting of temperatures, meter readings, and other quantities at regular intervals of less than a minute. Plotting of cooling curves for determining heat losses in calorimetry, exercises on Newton's law of cooling, and averaging voltage and current in a heating circuit when fluctuations render the use of single readings inaccurate, are but a few of the instances in which the use of an audible signal is much more convenient than that of a watch. The apparatus described in this note was developed to meet such needs and allow a student to keep his eye on the meter or thermometer right up to the instant of recording.

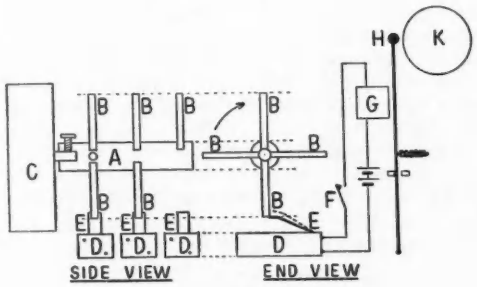

FIG. 1. Audible repeating time signal device.

The accompanying diagram shows the essentials of construction. A half-inch diameter brass rod A was drilled to take ⅛-inch brass arms B, about an inch long, in the form of radial spokes. One set of four spokes at quadrant intervals provides actuation of one timing switch, two at 180° intervals operate a second switch, a third single spoke operates a third switch. The rod was drilled to fit the driving stem of a standard 60-cycle clock movement C making one revolution per minute. The micro-type switches D, normally open but closed on depressing the actuating arm E, are mounted so that the spokes depress the arms and then release them at quarter, half, and whole minute intervals, respectively. Bat-handle switches F are wired in series with the micro-switches for selecting the time intervals desired. The armature G of a single-stroke dc electric bell is wired in series with a battery and in series with the paralleled microswitch circuits.

As soon as a microswitch is closed by the rotating spokes, provided the appropriate selector switch F is also closed, the armature of the bell is actuated and the gong striker H is pulled over, the sound thus produced serving as an advance warning of the termination of an interval. As the spoke releases the microswitch arm the armature is de-energized and the gong K is struck, giving sufficient sound to be heard easily throughout a large laboratory.

Quarter, half, and whole minute intervals have been found to satisfy the needs of this laboratory. A wider selection of fractional minute intervals may be provided by extending the spoke arrangements. For periods in excess of one minute, the actuating clock C may be replaced by one giving five minutes per revolution or even more. The device may be synchronized with other electric clocks within the laboratory so that the number of intervals employed may be checked.

Students have remarked on the convenience of this method of timing, which has been in use here for several years. The present model was constructed from war-surplus material at negligible cost. It should be possible to construct a similar one from standard supplies for a modest sum.

A Simple Method of Determining the Latent Heat of Steam

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THE errors involved in the determination of the latent heat of steam are well recognized but difficult to allow for in the conventional methods used in undergraduate laboratory work. The elaborate precautions taken by research workers to achieve reasonable accuracy[1] have no place in elementary work as they tend to cloud the mind of the student with a host of details which mask the essential principles involved. The method to be described, although not achieving the accuracy of the more elaborate methods, eliminates most of the errors without recourse to elaborate apparatus. It possesses the added attractions of ease of construction of apparatus and simplicity of operation.

The method is not entirely original but embodies several ideas already used by workers in this field, yet so adapted as to allow fairly accurate values of the latent heat of steam to be obtained even by freshmen. The apparatus is shown in Fig. 1. It employs an evaporation chamber similar to that originally used by Bertholet and improved upon by Kahlenberg.[2] A condenser is used as in the apparatus developed by Awbery and Griffiths[3] but serves to measure the rate of evaporation and not the rate of transfer of heat from boiler to receiver. The operation is dynamic and permits the elimination of errors which result from heat losses by the method of differences as explained below.

Construction of the apparatus is quite simple and can be carried out in any laboratory from standard equipment. A 2000-ml, short-neck, round-bottom flask is fitted with a medium-length condenser by means of a rubber stopper. The upper end of the condenser tube is arranged to reach to about one inch below the bottom of the inverted flask. A heating coil, mounted in the lower portion of the flask, provides the source of heat for the evaporation. This is made by winding about a yard of No. 30 nichrome wire in a close spiral on a one-eighth inch steel rod. The resistance so

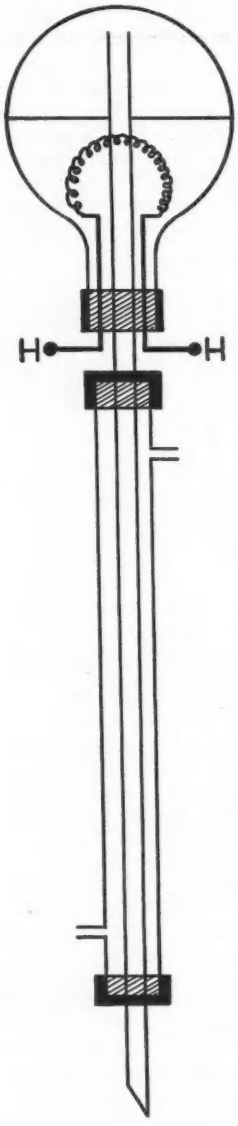

FIG. 1. Diagram of apparatus for laboratory determination of the heat of vaporization of water.

obtained is about 25 ohms and generates ample heat when operated on 120-v service. The coil, after the form has been removed and it has been pulled out somewhat to separate the turns, is secured to two stout copper leads passing through the rubber stopper which closes the flask. Water is introduced into the flask to the level shown before mounting the condenser. A good flow of cooling water in the condenser is essential to satisfactory operation.

To start a determination the water is heated up to boiling, the rheostat being then adjusted to produce as rapid an evolution of steam as possible without running the risk of driving water directly into the condenser. Most of the steam thus generated passes into the condenser and collects at a steady rate in the receiving flask when the temperature distribution of the apparatus has assumed a steady state. The balance of the steam condenses on the sides of the flask because of convection and radiation losses. Because of the convexity of the top of the flask and the clearance above the top of the condenser tube, the resulting water returns to the boiler and does not enter the condenser. If the rate of evolution of steam by the boiler is sufficient to produce distillate, the temperature distribution of the upper portion of the apparatus will remain practically the same whether boiling is rapid or gentle, steady conditions being assumed. The heat losses of the boiler, therefore, will be practically independent of the heat input and can be eliminated by the method of differences.

With steady conditions established, the distillate is collected in a weighed Erlenmeyer flask for a known period of time, say 200 seconds, reweighing giving the mass of steam evaporated per second. During the period of collection the voltmeter and ammeter are read simultaneously (by two operators) every ten seconds, the time for each pair of readings being indicated by an audible signal device which has already been described by the author in this journal.[4]

A second run is then taken in a similar manner using about one-third to one-half of the original input wattage, the appropriate setting of the control rheostat being found by trial.

On the assumptions discussed above we have:

$$\frac{V'A'}{4.18} = M'L + \text{heat losses},$$

$$\frac{V''A''}{4.18} = M''L + \text{same losses},$$

where V and A denote the averages of voltage and current, respectively, M the mass of water collected per second, and L the latent heat of steam in calories per gram, the superscripts referring to the first and second runs, respectively. By taking the difference the losses are eliminated and we find:

$$L = \frac{(V'A') - (V''A'')}{4.18(M' - M'')}.$$

It has been found necessary to correct the current as read by the ammeter for the current taken by the voltmeter, as the latter is greater than the error in reading the former. Six-inch mirror-scale Weston instruments are used in this laboratory, the voltmeter taking 75 ma at full-scale deflection of 150 volts. Table I shows a summary of the actual results obtained with this apparatus in a recent laboratory exercise.

Over a period of three years this apparatus has worked very consistently requiring less than an hour to obtain a result. The values have ranged from 530–540 as extremes, the bulk of them falling very close to the accepted value.

Refilling of the flask after each run is best carried out while the apparatus is still hot. By inverting the whole apparatus and pouring the required amount of cold water into the

Table I. Table of results: March 20, 1953.

Readings	Run No. 1	Run No. 2
Average voltmeter reading (volts)	115.4	70.0
Average ammeter reading (amperes)	4.12	2.50
Current taken by voltmeter (amperes)	0.05	0.03
Current in heating coil (amperes)	4.07	2.47
Mass of water collected in 200 seconds (g)	35.97	9.63
Result: $L=538$ calories/gram		

open end of the condenser, the added water is sucked into the flask by the contraction of the air produced by the general lowering of temperature. This procedure is much better than removing the stopper which tends to adhere to the flask after heating.

One source of error still remains. The steam carries over a small amount of spray which, by raising the value of M, tends to produce a lower value of L. Our results show that this source of error is small and is of the same order of magnitude as the errors in reading the meters. The method of Mathews[1] for eliminating this error (based on the assumption that the spray content is proportional to the rate of boiling) cannot, therefore, be applied here.

By carrying out the runs under reduced pressure, it should be possible to measure the latent heat at lower boiling points. Also, the latent heats of liquids other than water may be determined provided these liquids do not attack the rubber stopper. A more elaborate setup using tapered glass joints between the flask and condenser would eliminate the rubber stopper and thus extend the usefulness of the device. It is believed that the apparatus, even in its present form, may prove a useful addition to the average college laboratory.

[1] Smith, J. Opt. Soc. Am. **10**, 711 (1925); Smith, Phys. Rev. **33**, 173 (1911); Griffiths, Trans. Roy. Soc. (London) **A186**, 261 (1895); Mathews, J. Phys. Chem. **21**, 569 (1917); Henning, Ann. Physik **21**, 849 (1906); Henning, Ann. Physik **58**, 759 (1919).
[2] Kahlenberg, J. Phys. Chem., **5**, 215 (1901).
[3] Awbery and Griffiths, Proc. Phys. Soc. (London) **36**, 305 (1924).
[4] T. G. Bullen, Am. J. Phys. **21**, 645 (1953).

AMERICAN JOURNAL OF PHYSICS VOLUME 21, NUMBER 8 NOVEMBER, 1953

LETTERS TO THE EDITOR

Concerning Definitions of Magnetic Quantities

IN his article "Definitions of Magnetic Quantities," Dr. Hughes attempts to find a consistent interpretation of the magnetic vectors **B** and **H** without the necessity of first introducing dipoles.[1] It appears to this author that his treatment is open to several criticisms. His basic tenet resides in the assumption that magnetic fields are always produced by circulating currents and hence there is no need to speak of "fictitious dipoles." It is true that the rotation of electrons in their "orbits" may be considered as small circulating currents which produce magnetic fields; however the electron "spin" which almost solely contributes to ferromagnetism is only one step removed from the concept of "fictitious dipoles" which so abhors Dr. Hughes. Furthermore, it becomes very embarrassing when one tries to explain the enigma of the magnetic moment of a neutron (which has no net electric charge) in terms of circulating currents.

The fallacies of Dr. Hughes' arguments go still deeper. Having discarded the use of magnetic dipoles he then proceeds to establish a "basic" definition of **B** and **H** in terms of the "lines" of **B** and **H**. The entire burden of the description is thrust upon the mistaken concept that the lines of **B** and **H** possess greater reality than do dipoles, and by tracing the course of these lines one could supposedly ascertain all the properties of the field. This is an all too common error. First, "lines" have no objective reality. It is the position vector **B** or **H** which is experimentally measured and not the lines. Second the magnetic "lines" in general are not closed curves. Since the magnitude of **B** or **H** along those lines is not constant, there is no way, in general, of knowing in what direction to move in space so that one is assured of returning to the starting point. Together with this mistaken concept one could also demonstrate that pictures of "collapsing lines" and "cutting of lines" are equally fallacious.

Space does not permit a detailed discussion of Dr. Hughes' results, but a few of his points need clarification. On p. 95 he raises the question of whether **H** or **B** should appear in the circuital relation. Actually no question exists. If one integrates **H** around a closed curve the answer is always $\oint \mathbf{H}\cdot d\mathbf{l}=4\pi\Sigma i$, where i is the conduction current. Since that portion of **H** due to the magnetic dipoles of the media is derivable from a scalar potential, its integral around a closed path is always zero. The line integral of **H** due to conduction currents is, of course, the above formula. In the case of the integration of **B** around a closed curve, one always obtains $\oint \mathbf{B}\cdot d\mathbf{l}=4\pi\Sigma i+4\pi\int \nabla\times\mathbf{M}\cdot d\mathbf{S}$, since $\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$. On p. 91 Dr. Hughes makes the following statement concerning the fields due to a cylindrical magnetized bar. "If the bar is permanently magnetized, then **H** is in the same direction as **B** outside the bar, the change of direction of **H** occurring suddenly at the end faces of the bar. Such a sudden change of direction does not occur for **H** of the equivalent solenoid, and the method fails to explain the discrepancy." That there is no discrepancy is immediately evident from the following considerations: If one wishes to calculate the magnetic field due to both conduction currents and equivalent "Amperian currents," one evaluates the field from the vector potential **A** given below:

$$\mathbf{A}=\frac{1}{c}\int\frac{i+\nabla\times\mathbf{M}}{r}\,dv,$$

where $\nabla \times \mathbf{M}$ represents the equivalent "Amperian current" due to the magnetization of the medium. Furthermore only **B** is derivable from **A**, ($\mathbf{B} = \nabla \times \mathbf{A}$), in the general case. Hence it is the magnetic quantity **B** which is properly obtained from the "equivalent current" and is continuous across the boundary. Having obtained **B** one then calculates **H** at the boundary of the bar magnet, and it will show the proper discontinuity.

Finally, it is noted that Dr. Hughes apologetically "introduces poles as a convenient mathematical device, without physical significance." The meaning of "physical significance" is of course open to question. It seems to the author that modern physics is relying more and more heavily on mathematical structure for its interpretation of experimental data. If one obtains a mathematical formulation consistent with experiment, and the former yields such entities as "spinning electrons," "probability distribution," "magnetic poles," etc., one is not really perturbed. Reliance on "physical significance" *per se* is in essence a fruitless quest.

In conclusion it is worth noting that almost all the theoretical calculations in the modern theory of magnetism are carried out in terms of dipoles and polarization.

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[1] J. V. Hughes, Am. J. Phys. 21, 89 (1953).

A Discussion Concerning "Definitions of Magnetic Quantities"

THE letter[1] by Mr. Katz raises two separate and distinct issues; first, a philosophical discussion on the relative realities of "magnetic dipoles" and "circulating currents"; and second, clarification of some of the points raised in my original article.[2] These are considered separately below.

On the philosophical question, Mr. Katz' perplexities are due to a misapprehension of my intentions, which the wording of the original paper may well have fostered. I was following a train of argument which is commonly expressed in terms of the relative merits of atomic circulating currents and atomic dipoles, but which in reality depends on a preference for one method of expressing our fundamental results over another. I welcome this opportunity to put the argument in terms not involving the atomic models.

The older, and extremely clearly defined, method of procedure was first to define a "unit magnetic pole" in terms of the law of force between isolated poles; then to introduce the experimental fact that an electric current also produced a magnetic field, which was manifested by a force on an isolated pole; and then to use this experimental fact to establish a definition of unit current. The objections raised against this method were twofold: that it was a very roundabout, and experimentally untrustworthy route to get to a definition of our most important quantity, the unit current; and that it inferred that one and the same thing, a magnetic field, could be produced by two different and apparently unrelated agencies, a permanent magnet and a current-carrying wire. This is what is called, for convenience, the "fictitious dipole" method.

More recently, a method has developed in which the ampere is defined, *ab initio*, in terms of the forces between two current-carrying conductors. The law of force in this case can conveniently be split into two terms, the "flux density" produced by one of the currents, and the interaction of this flux density with the other current. This immediately circumvents the first objection to the older method, but it still leaves permanent magnetism to be explained. One can, of course, merely accept permanent magnetism as a separate phenomenon, retaining the second objection to the older method. The alternative is to try to explain permanent magnetism in terms of currents, which must almost automatically come down to "atomic circulating currents." This newer method has much to recommend it, but in the form of presentation commonly used it is sooner or later forced back to using dipole theory; and somewhere in the process a sleight-of-hand seems to occur, whereby the final result is one which is apparently incapable of explanation directly in terms of circulating currents. An example of this was given in the original article for the case of a bar magnet.

My purpose in writing the article was to improve the line of reasoning in the second method, so that explanations in terms of circulating currents remained in terms of circulating currents throughout, and so that the difference between the magnetic fields of a bar magnet and of a solenoid, for example, could be appreciated with the bar magnet's field described solely in terms of circulating currents.

From a purely philosophical viewpoint, I am willing to agree with Mr. Katz that the "atomic circulating current" is almost as fictitious as the "atomic dipole." That I recognized this is evident from the fact that I devoted an entire section of the original paper to a statement of what I meant by atomic circulating current. Having agreed with Mr. Katz on this issue, I hope he will agree with me that a theory which sets out to explain all magnetic phenomena in terms of currents should be able to do so without having to introduce atomic dipoles (however sound they may be philosophically) somewhere along the way.

Mr. Katz' second point is that some of the concepts of the original article are open to question, and some of the discussion deals with self-evident facts. I will take the questions raised in his letter, in order.

(a) His statement, that I gave a "basic" definition of **B** and **H** in terms of the "lines" of **B** and **H**, is completely incorrect. Section VII, which dealt specifically with the final definitions of **B** and **H**, did not include a single reference to lines. I hoped that I had made it clear that thinking in terms of lines was very helpful (to me, at any rate) in leading toward rigid definitions, but that the final definitions *must* be stated in practical terms, without reference to lines.

(b) On the question of whether lines of **B** and **H** possess greater reality than dipoles I am not prepared to argue. I have indicated above that I was trying to avoid mention of dipoles for reasons of consistency of a theory rather than of

"reality." With regard to his statement that there is no way, in general, of knowing in what direction to move in space to follow a line of **B** and **H**, perhaps Mr. Katz has never performed the elementary experiment of scattering iron filings on a glass plate over a magnet. The fact that magnetic "lines" in general are not closed curves was referred to in footnote 6 of the original paper; I fail to see that this fact in any way invalidates any of the arguments in the paper.

(c) Mr. Katz implies that it is self-evident that **H** rather than **B** should occur in the circuital relation; and he states that there is no discrepancy between the sudden field intensity changes at the face of a bar magnet and the absence of such changes at the ends of a solenoid. In both cases he proceeds to prove his point by introduction of a vector **M**, the intensity of magnetization of the medium. I have no objections to his treatment; I have said earlier, both here and in the original article, that the older method based on magnetic poles is complete, and logically sound. My point was that the newer theory, using currents exclusively, certainly had difficulty with these two points, and it is no assistance to the newer theory to say that the older theory met no such difficulties.

(d) Mr. Katz raises the question of the meaning of "physical significance," particularly as applied to dipoles. This is just a restatement, possibly on a wider basis, of the philosophical question discussed earlier, and it hardly seems worthwhile to discuss it further.

(e) In his conclusion, Mr. Katz states that nearly all the theoretical calculations in the modern theory of magnetism are carried out in terms of dipoles and polarization. It was never my intention to deny this fact, and the first sentence of Section VIII stated "It is not suggested that the methods outlined in Sections V–VII will replace the concept of magnetic poles, or dipoles, for purposes of calculation." But the fact that one prefers to use dipole theory in any particular problem, as a matter of convenience and because of the vast amount of theory already developed along those lines, is no reason for condemning a theory which avoids mention of dipoles. A parallel case would be to infer that, because all modern ac theory uses complex exponentials, one should never try to develop an ac theory using trigonometrical functions only.

In conclusion, I should like to thank Mr. Katz, and the Editor of this Journal, for giving me this opportunity to clarify the philosophical position, and to explain in more detail the reasons which led me to prepare the original paper.

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[1] H. W. Katz, Am. J. Phys. 21, 647 (1953).
[2] J. V. Hughes, "The definitions of magnetic flux density and field intensity," Am. J. Phys. 21, 89 (1953).

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ANNOUNCEMENTS AND NEWS

Mr. Tompkins Learns the Facts of Life. GEORGE GAMOW. Pp. 88+xi; Plates 6, Figs. 14, 17×22 cm. Cambridge University Press, New York, 1953. Price, $2.75.

Any lingering doubt which may exist as to the established position of biophysics may be considered dispelled now that that versatile dreamer, Mr. Tompkins, has turned his attention toward obtaining first-hand information concerning life processes. Except for one nostalgic involvement with binary arithmetic and a computer, these adventures are concerned chiefly with what goes on in the blood stream, the digestive system, and the brain. The lay reader (which in this case includes most physicists) may possibly be somewhat overwhelmed by the unfamiliar biological vocabulary—erythrocytes, leucocytes (alias phagocytes), maematin, chromosomes, adenosine-triphosphate, mitochrondria, and the like—and consequently find portions "rather interesting, but not too persuasive." At times Mr. Tompkins' guides have difficulty in simplifying the complex state of affairs which exist inside us, but they go at it manfully.

The modified plates are worth a special note of praise, for the additions to them contribute a delightfully humanizing note to the scientific dispassion of the photographs. The drawings, too, add much, with the exception of a rather confused one showing the distribution of solar energy, and one portraying the epinephrine molecule which dispenses with three of its hydrogen atoms. In connection with this latter, Mr. Tomkins' rapid counting to 22 atoms neither agrees with fact (26) nor figure (23). It is hoped that this will not come to the attention of his employers at the bank!

"The Professor's Lecture" which concludes the volume is devoted to a justification of "cutting in" on the field by physicists because of its importance to everyone—and a doubt as to the mathematical prowess of the biologist—and to the professor's concept of the nature of life as a physical process based upon the utilization of negative entropy. The professor himself has a little trouble with exponents in dealing with solar energy, and also makes the remarkable statement that a body does not radiate visible light when its temperature is below 800 degrees centigrade. Possibly his original figures were revised by the printers.

The correctness of the biological data and theory is a matter which has not been investigated by this reviewer, whose lack of biological information is deplorable. However, a smattering of Russian leads one to question the translation of the guide in the Land of Cerebrum, and to object to the transliteration of the words. Since the guide to the computer had Greek type available, the absence of

Russian type seems to represent either discrimination or excessive caution.

All in all, with biophysics rearing its head higher and higher, it seems likely that these adventures of Mr. Tompkins should be perused by physicists as a matter of self-defense, or at least self-interest. On this basis, the profession is indebted to Professor Gamow for having had time to record them to the benefit of our gnoceo-genes.

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Introduction to Solid State Physics. CHARLES KITTEL. Pp. 396+xiii, Figs. 171, 15.5×23.5 cm. John Wiley and Sons, Inc., New York, 1953. Price $7.00.

The great practical importance of solids has led in recent years to an increasing demand in industrial and government laboratories for scientists trained in the physics of the solid state, a demand that our educational institutions are as yet failing to fulfill. One important reason for this failure, doubtless, is the lack of elementary textbooks for instruction in this subject. Consequently, even a poor undergraduate text on solid state, if it should stimulate the beginner's interest in this subject, might prove worthwhile at this time. Since Professor Kittel's book is by no means a poor one, it will probably make an important contribution to the training of additional workers in this field. In particular, it is to be expected that a number of schools which do not now present courses in solid state, lacking a suitable text, will find it more convenient to do so with Kittel's book as a guide.

According to the preface, "This volume is intended as an introductory textbook in solid state physics for senior and beginning graduate students in physics, chemistry, and engineering." In a few sections, too much use is made of quantum mechanics for the presentation to be understood thoroughly by most seniors, but in general the level is not too advanced for physics seniors with a good background in atomic physics. The author sometimes omits the very elementary, although perhaps not obvious, explanations, so that, as implied in the preface, reading of the book should best be accompanied by more qualitative lectures. (There are 757 labelled equations in the 377 pages.)

The contents of the book (16 chapters) include most subjects which can be intelligibly discussed with little or no quantum mechanics and "in terms of simple, concrete and well-developed models," namely: crystal structure, ionic binding, elastic, thermal, dielectric, and magnetic properties, superconductivity, free electron and band theory, semiconductors, and imperfections. There are, in addition, 21 short appendices covering the more advanced details, results of which the author uses in his presentation. In a book as short as this the selection and weighting of subject material are largely matters of taste, and this reviewer can only state that his taste does not differ greatly from the author's. Although the book is written from the theoretical point of view and without description of experimental techniques, the many figures and tables comparing the results of theory with experimental data serve continually to indicate what kinds of measurement can be made, and to indicate the accuracy of both the measurements and the theory.

The principal criticism of this reviewer refers to the style of the first few chapters. Here the exposition is somewhat formal, and the pedagogical value would have been increased by the inclusion of more physical discussion. However, throughout the remainder of the book this formal tendency is admirably complemented by frequent order-of-magnitude estimates which point out the physical mechanisms of importance. These brief semiquantitative calculations, and the physical insight they give, represent one of the strongest points of this text.

While the book is not intended to be used at the research level, it will have some value as a starting point for beginners in the fields covered here. References are given to many of the classic papers, and in most cases the author indicates the directions of current research in these fields. The book is extremely up-to-date on most topics, and cites references to some of the latest theoretical papers even when the theories are not discussed in detail.

Further distinguishing characteristics of this textbook are the inclusion of 115 excellent problems, many of which require recourse to the current literature, and thorough author and subject indices. There are remarkably few errors and misprints for a first printing.

Although chemistry and engineering students without unusually good backgrounds in atomic physics will probably find this book difficult, the undersigned has no hesitation in recommending it for beginning graduate students and seniors well trained in physics. It is certainly to be desired that our colleges and universities will make use of it.

D. L. DEXTER
University of Rochester

Radiations and Living Cells. F. G. SPEAR. Pp. 222+xii, Figs. 60, 14×19 cm. John Wiley and Sons, Inc., New York, 1953. Price $3.50.

This volume from the *Frontiers of Science Series* was written "to occupy (an intermediate position) between a popular discourse, on the one hand, and a standard book for specialist readers on the other." This is a task at which Englishmen are particularly adept and to which they have committed many of their finest scientific minds. This report, from the leading British biological laboratory engaged in the study of radiation effects on living systems, is a good representative of such efforts.

The opening quotation is from *The History of the Royal Society* by THOMAS SPRATT (1635–1713), Lord Bishop of Rochester: "Some must gather, some must bring, some separate, some examine, and (to use a similitude which the present time of the year and the ripe fields that lie before my eyes suggest to me) it is in Philosophy as in Husbandry: Wherein we see that a few hands will serve to measure out and fill into sacks that Corn which requires very many more labourers to sow and reap and bind and bring into the Barn." And at the end, Spear quotes again from the

same paragraph with which he began: "If they (i.e., many men who enquire of experiments 'what solid good will come from thence') will persist in contemming all experiments except those which bring them immediate gain and a present harvest: they may as well cavil at the Providence of God that he has not made all the seasons of the year to be times of mowing, reaping and vintage."

This is indeed the crux of the matter. Radiation biology is just beginning a harvest whose full fruits will not be reaped without additional labors by radiation chemists in particular. Nonetheless, it is quite clearly time for most of us to see how the work is coming.

There are many interesting particulars. By 1922 (twenty-seven years after Roentgen) probably a "hundred radiologists had died from malignant disease due to radiation." (Today, in the U. S. alone, about two trained radiologists die each year from radiation-induced leukemias.) But more than ten *thousand* died from the radiation released by the two bombs over Hiroshima and Nagasaki.

The specific effect of radiation on three body systems takes up three chapters. These are devoted to the skin and alimentary canal, to the blood and circulatory system, and to the generative system. There is a chapter on "Radiation and Cancer" and a final chapter on the mechanisms which have been proposed to explain the observed effects.

Thus, *Radiations and Living Cells* is primarily a descriptive rather than an analytical book. Any teacher who demonstrates x-rays or radioactive isotopes should possess the basic information presented by Spear.

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The Scientific Adventure, Essays in the History and Philosophy of Science. HERBERT DINGLE. Pp. 372+ix, Figs. 5, 14×22 cm. Philosophical Library, Inc., New York, 1953. Price $6.00.

This is a fascinating book, and the review might be summarized in a short sentence: read it, you will enjoy it, and learn a great deal about the philosophical background of scientific research.

But this advice does not exhaust the matter and some of Professor Dingle's ideas should be briefly discussed. Most of the chapters were previously published in scientific journals but they take their full meaning by comparison, and the whole book clearly states the very original point of view of the author on the philosophy of science. The standpoint adopted commands a view of a very wide field, and shows questions concerning the relations of science with religion and the humanities in a new light. The significance of scientific investigation is considered nowadays from a point of view which is entirely different from the old classical one.

Science used to be regarded as the examination of an independent external material world which caused and shaped our experience. The present position is to regard science as an attempt to formulate the regularities in our experiences, which represent the primary data. We do not believe any more in an outside world existing independently of the observer. An observation requires a definite coupling between the outside world and the scientist, and represents a perturbation of this world. This change in the philosophical position of the scientist was forced upon him by experimental facts, but its many implications have not been exhausted and remain to be discovered progressively. In the first chapter, "The Missing Factor in Science," the author emphasizes the role that should be played by critics and philosophers in the discussion of basic problems of science. "For all its impregnable basis and the stability of its super structure, science is always essentially unfinished. . . . It is a process, stretching through time, in contrast with the instantaneous or eternal character of traditional philosophy." This suggests the need for a critical school, working within the scientific movement itself and performing the function which criticism has performed in literature. "Science has progressed like a ship with an all-powerful engine and no compass or rudder or steersman." A well-equipped school dedicated to the history and philosophy of science could play the role of critics, but this must be done by the scientists themselves. A critical effort within science "can make science self-conscious and aware of the significance of what it is doing in relation to other human activities."

According to this general program, the author devotes the first part of his book to historical essays, emphasizing the progressive change of fundamental ideas and its slow repercussions in different fields of scientific activity. Astronomy through the sixteenth and seventeenth centuries provides very interesting examples for such discussions. In a chapter on physics in the eighteenth century, we find a very curious discussion of Benjamin Franklin's invention of the lightning rod. The Royal Society had recommended the invention

> "But, after the breaking out of the American Revolution, Franklin was no longer regarded by many of the members in any other light than an enemy of England, and, as such, it appears to have been repugnant to their feelings to act otherwise than in disparagement of his scientific discoveries. Among this number was their patron, George III, who, according to a story current at the time, and of the substantial truth of which there is no doubt, on its being proposed to substitute knobs instead of points, requested that Sir John Pringle (President of the Royal Society) would likewise advocate their introduction. The latter hinted that the laws of nature were unalterable at royal pleasure; whereupon it was intimated to him that a President of the Royal Society entertaining such an opinion ought to resign, and be resigned accordingly."

Any similarity with recent events is purely accidental!

The second part of the book deals with the philosophical background of modern science. This is where the critical genius of the author really reveals its penetrating qualities. The chapters on "Science and the Unobservable," "The New Outlook in Physics," "Physics and Experience," "Time Measurement," "A Theory of Measurement,"

represent a most original contribution to scientific thinking. The author compares the point of view of Einstein, introducing relativity of space and time, with the "operational definitions" of P. W. Bridgman, and the "Uncertainty principle" of N. Bohr and W. Heisenberg, without forgetting to quote the Vienna Circle and R. V. Carnap. Everywhere he discovers a similar outlook, leading to the impossibility to believe in an external world unperturbed by our observations. But this modern standpoint is not universally adopted. Eddington criticized it violently, and it has not yet been used in modern textbooks, where the treatment of "dimensions" still proceeds according to old-fashioned methods. This is a point which Professor Dingle very carefully discussed in a series of papers in the *Philosophical Magazine* from 1942 on. He just summarizes his standpoint in the present book, and proves that the new outlook on science has not yet been completely understood nor used to its full value.

The book concludes with two very interesting chapters on "Science and Ethics," and "Science and Religion." Let us repeat again our advice: do not miss this fascinating book.

L. BRILLOUIN
Watson Laboratory

Practical Aids for Teachers of Physics

College and high school teachers of science and mathematics will be interested in a new Office of Education publication, "Education for the Talented in Mathematics and Science," prepared by Kenneth E. Brown and Phillip G. Johnson, Office of Education Bulletin 1952, No. 15, 34 pages. For sale by the Superintendent of Documents, U. S. Government Printing House, Washington 25, D. C. at 15 cents. Readers will not only be pleased to see that the importance of husbanding our highest intellectual talent is clearly recognized but may welcome some of the statements in this bulletin which indicate that present high school curricula are perhaps not now serving the best needs of the nation.

There is a new and useful book of problems covering the usual range of elementary physics and also some special subjects such as electronic measurements, theoretical mechanics, intermediate heat, intermediate acoustics, atomic and nuclear physics. The compiler is Professor Robert L. Weber, the Pennsylvania State College. The details: College Physics Review. 72 pages, $8\frac{1}{2}\times 11$ inches; 75 cents each, special prices for 10 or more, available in State College, Pennsylvania at W. B. Keeler Bookstore.

One hundred eight movies are listed under the title, Physics, in the *Educators' Guide to Free Films*, 13th Annual Edition, published by the Educators Progress Service, Randolph, Wisconsin. Many of these deal with applied physics. Teachers may be interested in consulting the volume in a library before planning to show any of these films. A few lines are printed below each title, summarizing the type of film and indicating the organization by which it was produced. Instructions are given in the volume for making appropriate requests for the free use of these films.

A Unit Test on Light for a College Course in General Physics

Contributed by WILLIAM T. WILKS, *State Teachers College, Troy, Alabama.*

The following type of objective test has been used successfully in testing students in first-year college physics. It has the advantage of covering a wide range of understandings and is easily scored. It does not evaluate the ability of students to solve problems successfully that involve a step-by-step analysis of physical principles, and should be supplemented by questions which do measure this skill.

Directions: To the ***left*** *of each statement circle the letters corresponding to the answers you accept as correct. From* ***none*** *to* ***all may*** *be circled.*

a b c d The speed of light a—was first measured about the middle of the nineteenth century; b—is approximately 1100 ft/sec in a vacuum; c—is approximately 186 000 mi/sec in a vacuum; d—is approximately 3×10^{10} cm/sec in a vacuum.

a b c d A 60-watt lamp is rated at 40 candle power. What is the luminous efficiency of the lamp? a—less than 7.5 lumens/watt; b—between 7.5 and 10.5 lumens/watt; c—more than 10.5 and less than 15.5 lumens/watt; d—15.5 or more lumens/watt.

a b c d If the lamp of the above problem is suspended 5 ft above a horizontal table, what will be the illumination on the desk top directly below the lamp? a—less than 1.0 ft-candles; b—1.1 to 1.5 ft-candles; c—1.6 to 2.0 ft-candles; d—more than 2.0 ft-candles.

a b c d As a distant object moves toward a convex mirror, the image a—moves toward the mirror; b—moves away from the mirror; c—increases in size; d—decreases in size.

a b c d The image formed by a camera lens must always be a—inverted; b—smaller than the object; c—beyond the focal point of the lens; d—closer to the lens than is the object.

a b c d Two convex lens of 10 and 25 diopters power respectively are placed in contact. The resulting lens combination would have a focal length of a—less than 5 cm; b—from 5 to 8 cm; c—more than 8 and less than 12 cm; d—12 or more cm.

a b c d The film used in a motion picture projector a—is always placed at the focal point of the projecting lens; b—is run "upside down" in the projector; c—is always placed between the focal point and twice the focal point of the projecting lens; d—is placed at exactly twice the focal length of the projecting lens.

a b c d In the laboratory a telescope is made from lenses of 10 cm and 50 cm focal length respectively; a—the 10 cm lens should be used as the eyepiece;

b—either lens may be used as the objective; c—the telescope will have a magnifying power of approximately 2.5; d—distant objects will appear inverted when seen through the telescope.

a b c d With respect to velocity a—all wavelengths of visible light travel at the same speed in a vacuum; b—all wavelengths of visible light travel at the same speed in water; c—all wavelengths of light travel at the same speed in glass; d—long wavelengths of light are refracted more than are short wavelengths when they pass from air to glass.

a b c d In a motion picture projector the film is located 12 inches behind the projecting lens, while the picture is thrown on a screen 30 ft away. If 16-mm film is used, how wide will the projected picture be (approximately)? a—40 mm; b—48 cm; c—1440 mm; d—the image size cannot be determined unless the focal length of the projecting lens is known.

a b c d Light will be refracted in passing from one transparent medium to another only if a—the light strikes normal to the surface which bounds the two media; b—the ray strikes at an angle to the boundary; c—the two media have different indices of refraction; d—the speed of light changes in going from one medium into the other.

a b c d A far-sighted person a—has eye lens which tend to focus light from distant objects behind the retina; b—has eye lens which focus some of the light in front of the retina, the rest of the light behind the retina; c—should wear converging spectacles to improve vision; d—should wear concave lens spectacles to improve vision.

a b c d The image formed by a plane mirror can never be a—inverted; b—larger than the object; c—closer to the mirror than is the object; d—real.

a b c d The wavelength of sodium yellow light is given as 5893 Angstroms; that of mercury green light as 5461 Angstroms; a—sodium yellow has a greater frequency (kilocycles/sec) than does mercury green; b—the value in Angstroms could be converted to cm by multiplying by 10^{-6}; c—when passed through a diffraction grating the yellow light would bend more than the green light; d—in passing through a glass prism the yellow light would be refracted more than would the green light.

a b c d The sky appears blue because a—of the interference of light waves in space; b—air molecules and fine dust particles scatter blue light more than they do other wavelengths of light; c—blue light passes through the atmosphere more readily than do other wavelengths of light; d—blue light is refracted at the top of the earth's atmosphere.

a b c d In sunlight a mixture of yellow and blue paint appears green. If this paint mixture was illuminated only by a mixture of monochromatic yellow and monochromatic blue light, the paint would appear a—white; b—black; c—green; d—yellow.

a b c d Fraunhofer lines have enabled man a—to learn much about the atomic structure of certain gases; b—to develop a complete understanding of the nature of light; c—to develop better means of navigation; d—to identify certain elements existing in the atmosphere of the sun.

a b c d Which of the following tend to substantiate the belief that light is a form of wave motion? a—light sometimes shows interference fringes; b—light follows the inverse-square law with respect to illumination; c—light bends when it passes through diffraction gratings; d—light can be polarized.

a b c d To secure the greatest increase in the focal length of a convex lens, the lens maker should a—increase the radii of curvature and use glass of higher index of refraction; b—increase the radii of curvature and use glass of lower index of refraction; c—decrease the radii of curvature and use glass of higher index of refraction; d—decrease the radii of curvature and use glass of lower index of refraction.

a b c d An absorption (dark line) spectrum a—is related to a phenomenon studied in sound called resonance; b—is related to a phenomenon studied in sound called forced vibrations; c—is produced when light from a glowing gas passes through a prism; d—is produced by white light passing through incandescent gases.

a b c d Spherical aberration in a concave mirror a—is the failure of all rays parallel to the principal axis to meet at the principal focus; b—is more common than is chromatic aberration; c—may be decreased by using only a small portion of a sphere as a mirror; d—may be corrected by using a parabolic mirror.

a b c d An object 1-inch tall is placed 5 cm in front of a spherical mirror. A real image is produced on a screen 25 cm from the mirror. a—we can be certain a convex mirror was not used; b—the image must be 5 cm tall; c—we can be certain the image is inverted; d—the focal length of the mirror is between 4 and 5 cm.

a b c d The index of refraction for a diamond is roughly 5/2. For glass it is roughly 3/2; a—the critical angle for glass and air would be greater than the critical angle for diamond and air; b—light travels approximately 75 000 mi/sec in a diamond; c—light travels approximately 1.7 times faster in water than it does in a diamond; d—when light light passes at an angle from water into a diamond the rays are bent away from the normal to the bounding surface.

a b c d The polarization of light a—may be produced by reflection of light at a definite angle from a glass plate; b—shows that light waves are transverse rather than longitudinal vibrations; c—is utilized in 3-D movies; d—is involved in the phenomena called double refraction.

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. 21, 485 (1953)].

Anderson, Lowell Leonard (*J*), University of Rochester, Rochester, N. Y.

Aplin, Charles Milton (*J*), Box 9273, University Station, Reno, Nev.

Baird, Hollis Semple, Northeastern University, Boston, Mass.

Bolling, N. F., 353 Donaghey Ave., Conway, Ark.

Brown, Robert Ray (*J*), 525 Vila St., Winona, Minn.

Bulliet, Mildred Williams, 1730 Oxford St., Rockford, Ill.

Burnstein, Ray Aaron (*J*), Department of Physics, University of Washington, Seattle 5, Wash.

Eshbach, Ovid Wallace, 201 Kenilworth Ave., Kenilworth, Ill.

Harris, James Malcolm (*J*), 6391 Sherwood Rd., Philadelphia 31, Pa.

Humphrey, Samuel S., 1605 Air Base Group, APO 406, c/o Postmaster, New York, N. Y.

Kessler, Myer Michael, 6 Frost St., Cambridge, Mass.

King, Hubert E. (*J*), 3 S. Mallory St., Batavia, Ill.

Krans, Roelf Luppo, Izaak Evertslaan 10, Arnhem, The Netherlands.

Lake, Robert Edgar, 215 Ryerson St., Brooklyn 5, N. Y.

Lewis, Harold Walter, 2307 Sprunt St., Durham, N. C.

Loeb, Leonard B., Department of Physics, University of California, Berkeley 4, Calif.

Morrison, Philip, Department of Physics, Cornell University, Ithaca, N. Y.

Spencer, William Turner (*J*), 219 Midland Ave., Wayne, Pa.

Stewart, Martin Dolan, 21 Elizabeth St., Glens Falls, N. Y.

Stoneham, Charles Ernest (*J*), 49 E. Quincy St., North Adams, Mass.

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Proceedings of the American Association of Physics Teachers

The Pittsburgh Meeting, June 25–27, 1953

THE June meeting of the American Association of Physics Teachers in Pittsburgh, Pennsylvania, was in the informal, pleasant tradition of the summer meetings of other years. An interesting program, tours to points of interest in the Pittsburgh district, and much informal discussion with friends kept members and their families occupied for three days. Not the least of the attractions of the meeting was its location in the air-conditioned auditorium of Mellon Institute for Industrial Research. The sessions were held in comfort in pleasant surroundings although outside temperatures reached the low nineties on several days.

The program, arranged by President-elect Marsh W. White, included symposia on four problems of current interest to physics teachers, three invited papers, and three groups of contributed papers. On Thursday morning, June 25, Dr. Francis Young of the Committee on International Exchange of Persons, discussed exchange professorships for physics teachers. The teaching positions offered in exchange programs have had little attraction for U. S. physicists in contrast to the research positions which have been eagerly sought. In another reminder of lost opportunity, Professor T. D. Phillips of Marietta College pointed out that most physics departments organize their courses around the physics major and do little for the nonspecialist. He suggests that courses in musical physics, color, atomic energy and its control, and science fiction would attract students and give physics a more liberal reputation on the campus. A group of contributed papers closed this session.

Thursday afternoon was devoted to a symposium on the relation of physics to medicine with particular emphasis on premedical education. Dr. Paul L. McLain of the University of Pittsburgh School of Medicine presented the opinion that many students come to medical school with a poor preparation in physics because they are not strongly motivated to study physics as undergraduates. Dr. McLain went on to mention a number of applications of basic physics to medicine, any one of which could be used in a premedical physics course to convince students that physics would be useful to them. Dr. Martin Hanig, a research biophysicist at the University of Pittsburgh, gave examples of how physics is

essential in medical research, in particular in the investigation of atherosclerosis. Dr. E. L. Harrington of the University of Saskatchewan, who was unable to attend the meeting, discussed the premedical physics course in a paper read by Dr. R. Resnick. Dr. Harrington presented convincing arguments for a separate physics course for premedical students, one which would draw upon medical examples instead of the usual engineering ones. Discussion was lively and had to be terminated so that members could have refreshments in the Social Room of Mellon Institute as guests of the University of Pittsburgh.

Buhl Planetarium was host to members and their families on Thursday evening with a program that included the sky show and a special demonstration of the Zeiss projector.

On Friday morning, a symposium on the integration of high school and college teaching of physics presented four views of this important problem. Dr. Ralph W. Lefler, Purdue University, described what can be accomplished in improving the preparation of high school teachers of science by cooperation between training institutions, local administrators, and state administrators. Dr. J. M. Daniels, Carnegie Institute of Technology, discussing entrance requirements in engineering schools, pointed out that physics majors, although small in number in most institutions, are academically a hardy group, a large percentage of them successfully completing their four-year curriculum. Dr. Alfred Beattie, Superintendent of Schools in Allegheny County, Pennsylvania, reviewed the decline of physics enrollments in high schools during the past twenty years and discussed administrative steps that are being taken to reverse this trend. Finally, Dr. Harold F. Bernhardt, Temple University, described an active program being carried on by the Science Council of Philadelphia to get students interested in science by means of projects and to help able students obtain scholarships. Lengthy discussion indicated that this subject has great interest for members of the Association.

The session on Friday afternoon was opened by a symposium on transistor physics which was unusually well attended. Dr. C. Zener, Westinghouse Research Laboratories, reviewed the basic physics of semiconductors, particularly junction type transistors. Dr. S. J. Angello, Westinghouse Materials Division, discussed the "Unipolar" transistor and showed that modulation of semiconductor resistance with small power input can result in power gain. Dr. W. J. Leivo, Carnegie Institute of Technology, described six experiments, originally devised at Bell Laboratories, which he has used successfully to teach transistor physics. Dr. J. R. Haynes, Bell Telephone Laboratories, gave a number of interesting demonstrations of transistors, showing their versatility and compactness. Contributed papers closed this session.

Members and their families met for dinner on Friday evening at Hotel Webster Hall. Some pleasant and witty anecdotes related by President Paul E. Klopsteg and a violin and piano sonata played by two students of the School of Music at Carnegie Institute of Technology fittingly closed a leisurely meal. The group then moved to Mellon Institute auditorium for an open meeting of the Association. Dr. Klopsteg introduced Dr. R. E. Gibson, of the Applied Physics Laboratory at Johns Hopkins University, who spoke on "Science, Art, and Education." It would do scant justice to Dr. Gibson's elegant paper to attempt to summarize it in a few lines. It is hoped that it will be published in its entirety.

On Saturday morning, high school-college relations were again under consideration in a symposium on admission with advanced standing. The first three speakers have been associated with groups studying a proposal to strengthen the high school curriculum for exceptionally able students. Dr. William H. Cornog, Cooperative Committee on the Teaching of Science and Mathematics, discussed the philosophy of this approach, which is, in brief, to make the high school work more effective so that the able student can accelerate in college. Dr. Richard M. Sutton, Haverford College, listed topics in general physics which his group feels must be covered by a student in order for him to succeed in an advanced standing examination in physics. Dr. Morris Meister, Chairman of the Cooperative Committee, discussed the administrative problems associated with this plan. Finally, Dr. Bernard Watson, United States Department of

Labor, reviewed the other programs sponsored by the Ford Foundation including the pre-induction plan.

Five contributed papers were presented on Saturday afternoon. As a closing event, members and their families visited the Carnegie Institute of Technology Nuclear Physics Laboratory at Saxonburg, Pennsylvania. A ride through the farm country of Western Pennsylvania, an inspection trip to the synchrocyclotron laboratory, and a picnic dinner furnished the combination of physics and sociability which had characterized the entire meeting.

The symposia were recorded on magnetic tape by Professor J. G. Winans of the University of Wisconsin. The recordings may be borrowed by writing to him.

Fourteen publishers exhibited textbooks at this meeting, and three manufacturers of equipment displayed new demonstration and laboratory apparatus.

The local committee for the meeting included T. M. Donahue, University of Pittsburgh; R. C. Hitchcock, Buhl Planetarium; A. J. Kozora, Duquesne University; W. J. Leivo, Carnegie Institute of Technology; Charles Williamson, Carnegie Institute of Technology; and W. C. Kelly, University of Pittsburgh, chairman. Tours to laboratories, industries, and other places of interest in the Pittsburgh district were arranged by this committee and brought the usual tangled problems of transportation and scheduling. Members and their wives who attended the meeting were so appreciative of these and other arrangements, however, that the local committee feels well repaid.

The program committee consisted of R. L. Brown, Allegheny College; R. C. Hitchcock and R. N. Jones, Philadelphia School of Pharmacy and Science; W. C. Kelly; A. K. Van Scoy, Potomac St. School of West Virginia University; Wayne Webb, Pennsylvania State College; C. Williamson; R. M. Woods, Westminster College; and Marsh W. White, Pennsylvania State College, Chairman.

W. C. KELLY
University of Pittsburgh

Invited Papers and Symposia

Thursday Morning, June 25
R. C. COLWELL, Presiding

International fellowships and professorships. FRANCIS YOUNG, Committee on International Exchange of Persons.

Physics in the liberal arts college. T. D. PHILLIPS, *Marietta College.*

Thursday Afternoon, June 25
W. C. KELLY, Presiding
Symposium: *Physics in Medicine*

Physics in physiology. PAUL L. MCLAIN, Professor of Physiology, *University of Pittsburgh,* School of Medicine.

Applications of physics to medical research problems. MARTIN HANIG, Biophysics Department, *University of Pittsburgh.*

Premedical courses in physics—advantages and present trends. E. L. HARRINGTON, *University of Saskatchewan.*

Friday Morning, June 26
A. J. KOZORA, Presiding
Symposium: *The Integration of High School and College Teaching of Physics*

Certification of physics teachers. RALPH W. LEFLER, *Purdue University.*

College entrance requirements in mathematics and physics. JOHN DANIELS, *Carnegie Institute of Technology.*

Administrative problems in high school physics and mathematics. ALFRED BEATTIE, Superintendent, *Allegheny County School System.*

Guidance programs in high school and the stimulation of interest in physics. HAROLD F. BERNHARDT, *Temple University.*

Friday Afternoon, June 26
CHARLES WILLIAMSON, Presiding
Symposium: *Transistor Physics*

The physics of semiconductors. C. ZENER, *Westinghouse Research Laboratories.*

The physics of transistors. S. J. ANGELLO, *Westinghouse Materials Engineering Laboratory.*

Teaching applications of transistors. W. J. LEIVO, *Carnegie Institute of Technology.*

Some fundamental experiments in transistor physics. J. R. HAYNES, *Bell Telephone Laboratories.*

Friday Evening, June 26
PAUL E. KLOPSTEG, Presiding
Open Meeting Address

Science, art, and education. R. E. GIBSON, Director, Applied Physics Laboratory, *Johns Hopkins University.*

Saturday Morning, June 27
MARSH W. WHITE, Presiding
Symposium: *The School and College Study of Admission with Advanced Standing*

Philosophy and objectives of the study. WILLIAM H. CORNOG, Executive Director, The Cooperative Committee on the Teaching of Science and Mathematics.

Curricular modifications proposed in the field of physics. RICHARD M. SUTTON, *Haverford College.*

Administrative problems and arrangements. MORRIS MEISTER, Chairman of the Cooperative Committee.

Other Ford Foundation programs concerned with high school-college integration. BERNARD WATSON, *United States Department of Labor.*

Contributed Papers, with Abstracts

1. A direct experimental method of measuring the ampere in absolute units. DONALD S. AINSLIE, *University of Toronto.* Two types of current balances with parallel conductors were designed for the measurement of the ampere. The first type, employing an adjustable frame for the variation of the distance between the conductors, was used to give an experimental derivation of the Biot-Savart Law. The second type, with an arrangement to vary the length of the fixed conductors, was employed to determine the variation in the force with the length. These results were combined with the definition of the ampere to give a direct experimental approach to the problem of the measurement of the ampere in absolute units.

2. Projection oscilloscope. D. L. BURK AND T. H. FIELDS, *Carnegie Institute of Technology.* A projection oscilloscope, an adaptation of a Norelco projection television unit, was demonstrated. Frequency response, trace intensity, sweep speed, and picture size were discussed, and representative displays were shown. At an eight-foot projection distance the picture circle is five feet in diameter and the trace is one-quarter inch wide. A safety device interrupts the electron beam when there is danger that the target of the projection tube might be burned by a stationary spot.

3. Demonstrations with the Bragg model of a crystal lattice. FREDERIC KEFFER, *University of Pittsburgh.* A crude replica of the Bragg-Nye bubble-raft model of a crystal lattice[1] is very easy to make and use for demonstrations of dislocations and other crystalline imperfections. It is suggested that the demonstration could be given after students have become familiar with Hooke's law and the general nature of elastic constants, in order to make clear the manner in which dislocations allow for low elastic limits in solids. The type of soapy solution used by child bubble blowers and available at almost any dime store is quite adequate for the raft. One also needs a piece of glass tubing narrowed to a small opening and connected by a mechanical valve (to cut down air supply further) to a source of air. For uniform bubbles one must hold the opening of the glass tube several millimeters below the surface of the soapy solution. Finally one needs a projector and screen.

[1] W. L. Bragg and J. F. Nye, Proc. Roy. Soc. (London) **A190**, 474 (1947).

4. Oscilloscope pictures of intermodulation distortion. RICHARD C. HITCHCOCK, *Buhl Planetarium.* One low- and one high-frequency sinusoidal signal are simultaneously applied to the input of an audio amplifier. When the high-frequency component in the output is isolated, and shown on an oscilloscope, it is usually seen to be modulated by the low-frequency component. This is intermodulation distortion, imd. As the input voltage is increased, at first the imd increases at a faster rate, then tapers off. A multi-line graph, in addition to the oscilloscope picture, is needed for a reliable indication when the imd is greater than five percent.

5. An application of the idea of group velocity. E. MENDOZA, *Carnegie Institute of Technology.*

6. A note on the study of uniformly accelerated motion. HENRY S. C. CHEN, *Drexel Institute of Technology.* A new method of data analysis for the experiment on uniformly accelerated motion is described. There is no substitution of average velocity for instant velocity, nor is there any assumption that the acceleration is constant. The student is able to see in action the mathematical concept of a limit, namely, the instantaneous velocity is the limit approached by the average velocity as the time interval approaches zero. A typical set of data and the related curves were shown.

7. Kerr cell photography at exposures shorter than one microsecond. R. v. HEINE-GELDERN, *Carnegie Institute of Technology.* By synchronizing an electro-optic shutter (speeds to 2×10^{-7} sec) with an intense light flash (electrically exploded wire, peak intensity 5×10^{8} candle power), visible light photographs of a number of high-speed explosive phenomena have been obtained for the first time. The experimental procedures were described in some detail, and a few unusual results were presented.

8. The Geiger-Müller counter in the general physics laboratory. WALTER EPPENSTEIN, *Rensselaer Polytechnic Institute.* We have introduced experiments on nuclear physics during the sophomore year at Rensselaer Polytechnic Institute. This has been done for a number of reasons. They illustrate some of the basic principles of radioactive decay. They also make the student familiar with detection instruments, such as the Geiger-Müller counter and modern scaling equipment, which are becoming more and more important in many branches of engineering. In connection with experiments on nuclear physics the students are made aware of the dangers involved in the handling of radioactive isotopes. In the evaluation of their data they meet new situations, such as statistical fluctuations the result of random phenomena and the use of semilogarithmic graph paper. The experiments actually performed are standard experiments found in almost any junior or senior atomic

physics laboratory. The coefficients of absorption of lead and aluminum are compared, the range of beta rays in aluminum is determined, and indium is used for measuring the half-life time of an artificially radioactive substance. We feel that these and other experiments on nuclear physics should no longer be included in the curriculum just for the benefit of physics majors alone, but should be performed by all engineering students because of the increasing importance of nuclear problems in the world today.

9. Unifying concepts in engineering physics. F. T. ADLER, *Carnegie Institute of Technology.*

10. Some teasers for conclusion jumpers. RICHARD M. SUTTON, *Haverford College.* Teaching the able student not to jump to false conclusions may be accomplished in part by "teasers" that offer a physical situation to be diagnosed wherein the obvious answer may be wrong. Several examples are given: (1) The "falling chimney" apparatus previously described by the author[1] may be made to *fail* and the cup will not catch the ball if a 500-g weight is attached by a rubber band just behind the cup; thus, an added weight which might be expected to speed up the falling arm actually slows it to the extent that the experiment fails. (2) Simple considerations of momentum and energy show that, in an elastic collision between a rapidly moving club, such as a golf club, and a stationary ball, the maximum speed attainable by the ball cannot exceed twice the speed of the club, no matter how heavy the club. What is the maximum speed attainable by a *pitched* ball, if the ball's speed is V_1 and the speed of the bat is V_2? Offered to tease the student are six choices: $2(V_1 \pm V_2)$, $2V_1 \pm V_2$, $V_1 \pm 2V_2$. (3) If a particle A of mass 2 with initial velocity along the X axis strikes a particle B of mass 1 at rest at the origin and, after collision, the projectile particle A is deflected through an angle ϕ, is the direction of path of the struck particle B uniquely determined? Most people who are familiar with the vector triangle of momenta involved answer "yes." But with care, it becomes evident that two solutions are possible, both of which allow conservation of energy and momentum. (4) If a wire is wrapped just once around the base of a right circular cone, the same wire will go just twice around the same cone half way to its vertex. At some point between these two levels, the ends of the wire must be farthest apart as measured along a straight line connecting them. Where is this point? Simple considerations suggest that the ends are separated by a diameter of the cone when the wire goes $1\frac{1}{2}$ times around. This is not so!

[1] R. M. Sutton, *Demonstration Experiments in Physics* (McGraw-Hill Book Company, Inc., New York, 1938), Experiment M-206.

11. A note on the design of the spherometer. HENRY S. C. CHEN, *Drexel Institute of Technology.* A study of errors is made for the case of the spherometer. It is shown that the centering of the micrometer screw with respect to the base circle is not critical, and that the radius of the base circle should be measured with greater precision. It is hoped that the suggested new design will mean increased accuracy and easier construction.

12. Physics in Negro colleges, a progress report. DONALD A. EDWARDS, *Lincoln University,* (Jefferson City, Missouri). Course offerings in physics in the average Negro college follow the usual pattern of introductory courses, intermediate courses, and upper division courses. When compared with the average pattern of ten colleges whose students are predominantly white, and which have enrollments of one thousand or less, there are no significant differences in offerings, except that there are fewer courses in modern physics, electronics, mathematical physics, and radiation physics. Of every nine Negro colleges, five offer a major in physics, three offer only a minor, and one offers neither a major nor a minor. The deficiencies of the average Negro student entering the beginning course are of the same kind as those of any other student, but they are perhaps greater in degree. The three main deficiencies are in elementary mathematical background, in writing and reading the English language, and in mental disciplines. When considering only the Negro colleges which offer a major in physics, the comparison is very favorable with respect to adequate laboratory equipment, funds for equipment, and research facilities for the instructor. Unfortunately, however, the output of physics majors by Negro colleges is only about five per thousand students enrolled. This number is about one-half the number produced in other colleges.

13. Doppler effect equation. J. G. WINANS, *University of Wisconsin.* A single easily remembered equation which describes all cases for the Doppler effect is $f_o/(V-v_o) = f_s/(V-v_s)$, where f_o and f_s are the frequencies for observer and source, respectively, v_o and v_s are the components parallel to the line between observer and source of the velocities of observer and source respectively, and V is the velocity of sound. In using this equation the direction of V at the observer is taken as the positive direction and signs are given to v_o and v_s according to their directions. If there is a wind of velocity V_w the equation still applies if $V = V_w + V_{\text{sound}}$ is that vector sum of wind and sound velocities which is parallel to the line between observer and source. Thus, one equation and one rule of signs describes all cases for the Doppler effect.

14. A new examination of the laws of thermodynamics. ROBERT B. GREEN, *Stevens Institute of Technology.* The first law is stated in a new form not dependent on the Joule experiment: Any system which undergoes a cycle and exchanges *either* net heat or net work with its surroundings must exchange *both*. The fixed proportionality of net heat to net work is proven as a corollary. A statement of the second law is given. It is shown that any statement of the laws not using cycles depends upon an implied definition of a finite state, which probably cannot be made rigorously.

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